Checking Timed Regular Expressions on Boolean Signals

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Here we begin

Introduction

- Patterns exist on signals. (Intentionally or unintentionally)
- Checking patterns is an important verification task.
- Regular Expressions (Usual solution)
- Uses: PSL, SVA

This work

- A complete solution for Timed Pattern Matching problem

Timed Pattern Matching

Problem (Timed Pattern Matching)

Let $\mathbb{T} = [0, d]$. *Given a dense-time signal* $w : \mathbb{T} \to \mathbb{B}^m$ *and a timed regular expression* φ, *find all intervals* $(t, t') \in \mathbb{T} \times \mathbb{T}$ *that matches* φ.

We need to define ...

1. Timed Regular Expressions
2. Match-Set

Timed Pattern Matching

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Timed Regular Expressions

Definition (Syntax of TRE)

$$\varphi := \epsilon \mid p \mid \overline{p} \mid \varphi \cdot \varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi^{*} \mid \langle \varphi \rangle_{I}$$

(p propositional variable; I duration constraint)

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It will match ...

$\varphi := p$	—	any interval s.t. p uniformly holds
$\varphi := p \cdot q$	—	any interval s.t. p followed by q
$\varphi := \langle p \cdot q \rangle_{[3,4]}$	—	any interval with a duration between [3,4] s.t. p followed by q

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Definition (Semantics of TRE)

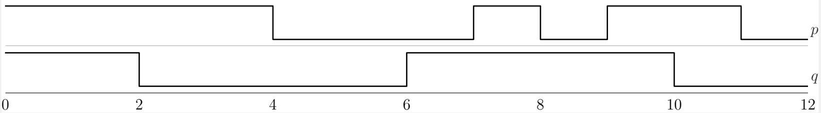
$$
\begin{array}{lll}
(w, t, t') \models \epsilon & \leftrightarrow & t = t' \\
(w, t, t') \models p & \leftrightarrow & t < t' \text{ and } \forall s \in (t, t') : \; p[s] = 1 \\
(w, t, t') \models \overline{p} & & \text{similar} \\
(w, t, t') \models \varphi \cdot \psi & \leftrightarrow & \exists t''. \, (w, t, t'') \models \varphi \text{ and } (w, t'', t') \models \psi \\
(w, t, t') \models \varphi \vee \psi & \leftrightarrow & (w, t, t') \models \varphi \text{ or } (w, t, t') \models \psi \\
(w, t, t') \models \varphi \wedge \psi & & \text{similar} \\
(w, t, t') \models \varphi^* & \leftrightarrow & \exists k \geq 0. \, (w, t, t') \models \varphi^k \\
(w, t, t') \models \langle \varphi \rangle_I & \leftrightarrow & t' - t \in I \text{ and } (w, t, t') \models \varphi
\end{array}
$$

An Example

Expression

$\varphi := \langle (p \wedge q) \cdot \bar{q} \cdot q \rangle_{[4,5]} \cdot \bar{p}$

Signals



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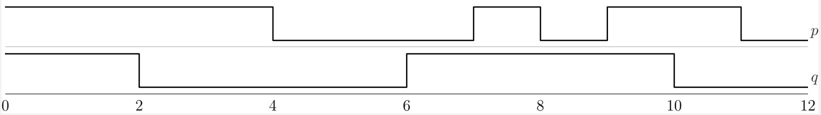
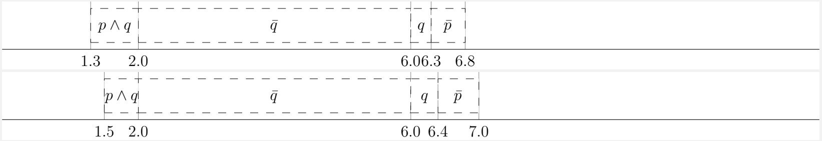


A few matches



Problem Statement

Definition (Match-set)

For a signal w and an expression φ the match-set is

$$\mathcal{M}(\varphi, w) := \{(t, t') \in \mathbb{T} \times \mathbb{T} \mid (w, t, t') \vDash \varphi\}$$

Problem (Timed pattern matching)

Given a signal and an expression compute the match-set.

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Data Structure

Definition (2D Zone)

A 2D zone is a subset of $\mathbb{R}^2$ described by inequalities

$$\underline{b} < t < \overline{b} \qquad \underline{e} < t' < \overline{e} \qquad \underline{d} < t' - t < \overline{d}$$

with $\underline{b}, \overline{b}, \underline{e}, \overline{e}, \underline{d}, \overline{d}$ are constants.

About 2D zones

- Representing a set of intervals (t, t')
- $[\underline{b}, \overline{b}], [\underline{e}, \overline{e}], [\underline{d}, \overline{d}]$ correspond begin, end and duration constraints.
- Zones (in general) used for timed automata verification; efficient algorithms and libraries exist.
- Many examples below.

Main Result

Theorem

The match-set $\mathcal{M}(\varphi, w)$ is a finite union of 2D zones. It is computable knowing expression φ and signal w.

Method for algorithms

Structural induction over φ.

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Method for algorithms

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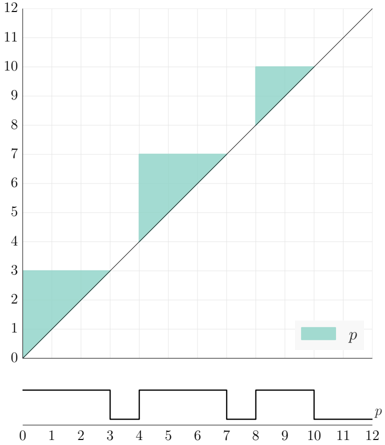
Computation: Literals

A Literal

$\mathcal{M}(p)$ — a union of triangle zones

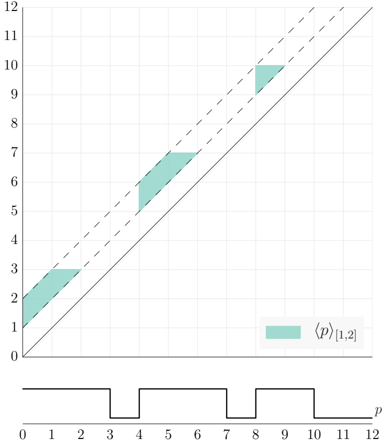


[1] Dropping w when w is clear

Computation: Time Restriction

Time Restriction

$$\mathcal{M}(\langle\varphi\rangle_I) = \mathcal{M}(\varphi) \cap \{(t, t') \mid t' - t \in I\}$$



Computation: Concatenation

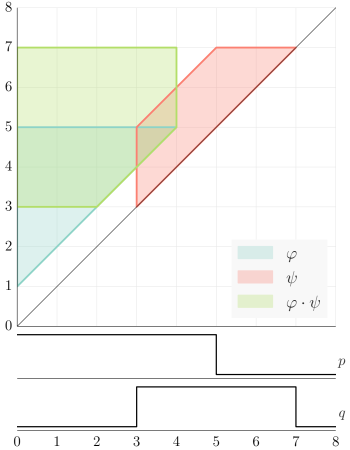
Concatenation

$\mathcal{M}(\varphi \cdot \psi) = \mathcal{M}(\varphi) \circ \mathcal{M}(\psi)$

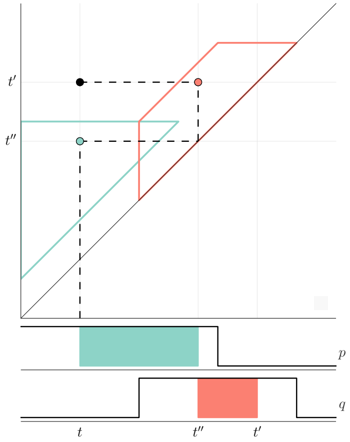
Composition preserves zones

- $(t, t') \in \mathcal{M}(\varphi) \circ \mathcal{M}(\psi) \leftrightarrow \exists t'' : (t, t'') \in \mathcal{M}(\varphi) \wedge (t'', t') \in \mathcal{M}(\psi)$
- Can be obtained using standard zone operations.

Concatenation with pictures



$\varphi := \langle p \rangle_{[1,\infty]}$, $\psi := \langle q \rangle_{[0,2]}$ and $\varphi \cdot \psi$



Explanation

Back to the First Example

Expression

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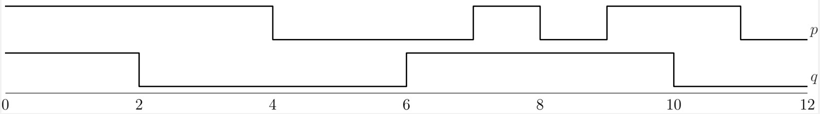
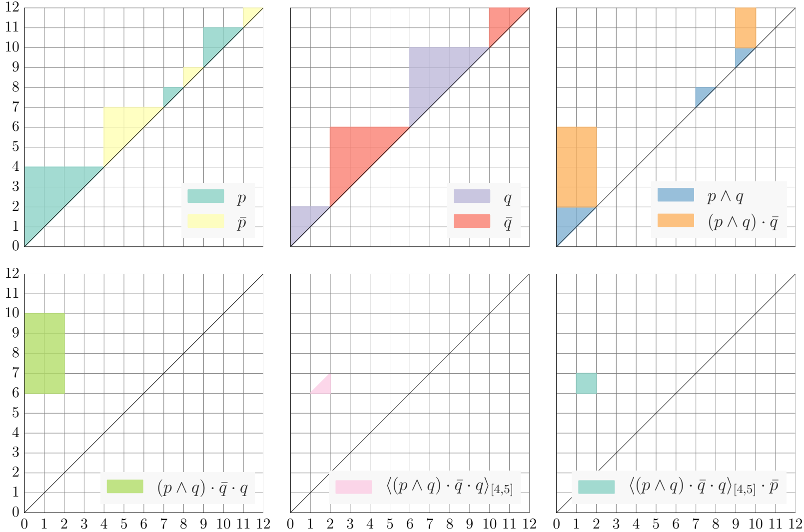


Correct Result

$\mathcal{M}(\varphi, w) := \{(t, t') \in [1,2] \times [6,7]\}$

Back to the First Example

Match-Set Computation



Performance

Experimental Setup

- A complex expression φ.
- Random signals p and q with variability $\mathcal{V}$ and length $\mathcal{L}$

Notes

- Python calling IF zone library (in C)

$$\varphi := \langle (\langle p \cdot \bar{p} \rangle_{[0,10]})^* \wedge (\langle q \cdot \bar{q} \rangle_{[0,10]})^* \rangle_{[80,\infty]}$$

$\mathcal{V}$	$\mathcal{L}$	$\lvert Z_\varphi \rvert$	Time (s)
0.025	40000	0	0.08
0.025	80000	0	0.17
0.025	160000	0	0.37
0.05	40000	0	0.27
0.05	80000	0	0.60
0.05	160000	0	1.27
0.075	40000	1	0.64
0.075	80000	4	1.40
0.075	160000	5	2.88
0.1	40000	10	1.35
0.1	80000	23	2.73
0.1	160000	47	5.83

Conclusion

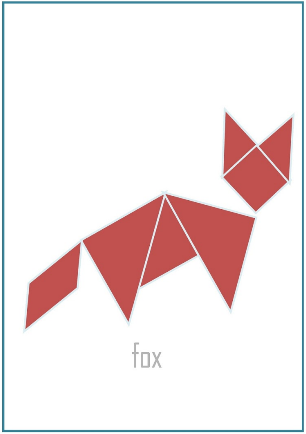
Summary

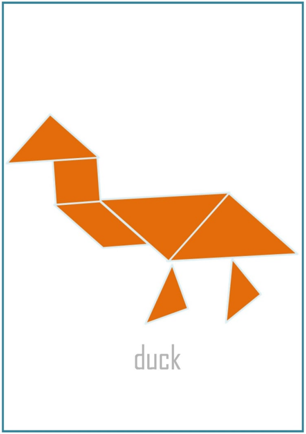
- Timed Pattern Matching
- Zones to represent match-sets
- Experiments witness scalability.

Discussion

- Trivial to extend for any discrete value domain.
- Natural companion for specification logics.

More patterns using zones





Thanks