Towards Formal Verification of Analog/Mixed-Signal Systems: „The Algebraic Approach“

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AG Design of Cyber-Physical Systems

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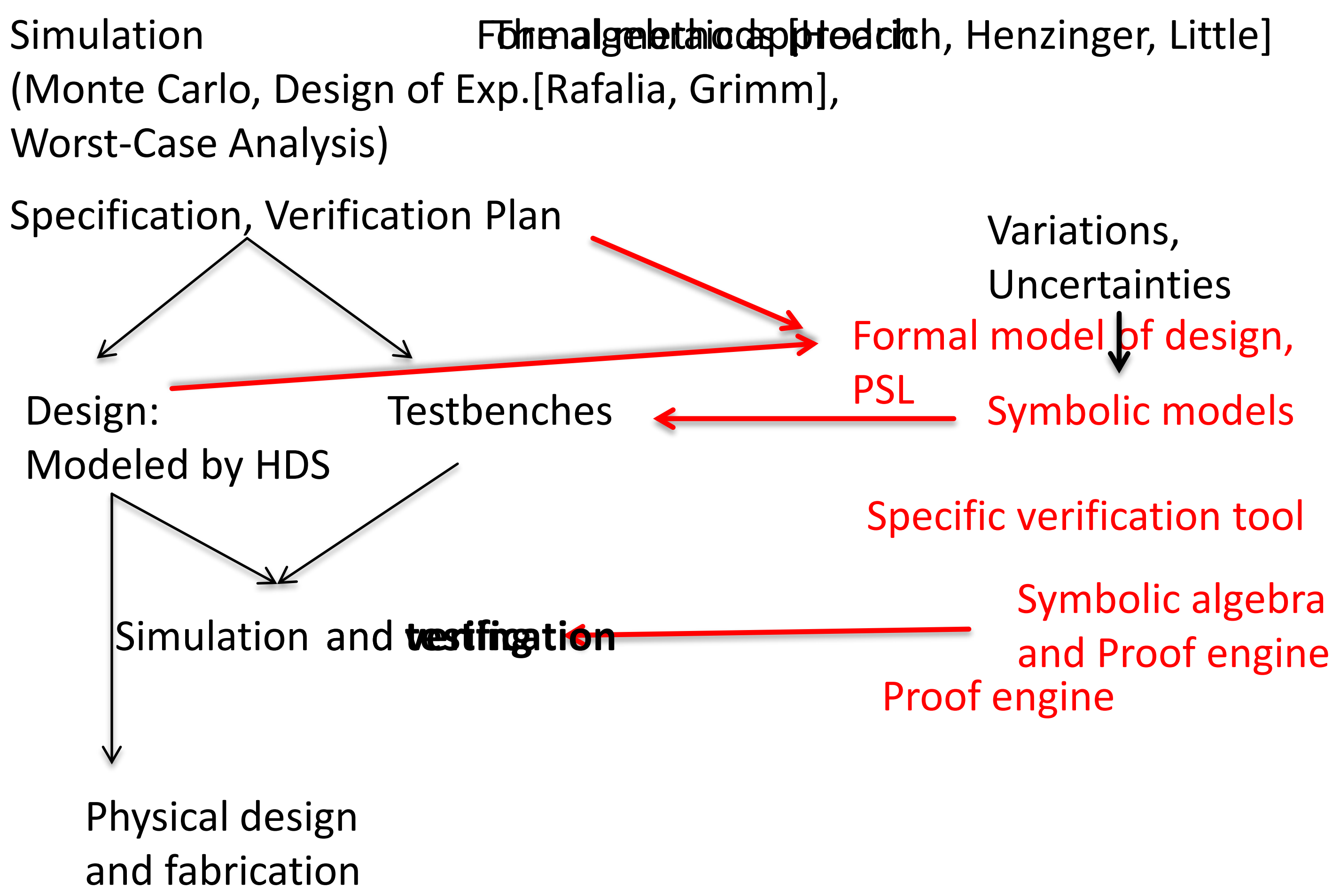
Digital v.s. AMS Verification

	Digital	AMS
Specification	Finite state automata	Math.functions (multipliers, integrators, differentiators, …,) + Properties (S/N, Step Response, BIBO stability)
Verification (To be covered)	Huge input space No variability	few measurements (not all possible inputs) Uncertainties/deviations ← challenge

Agenda

- Overview of approaches
- Related work: Affine Arithmetic
- Extended Affine Arithmetic for Systems with Discontinuities
- Benchmark: 3rd Order Sigma-Delta Modulator
- Conclusion

Overview of approaches

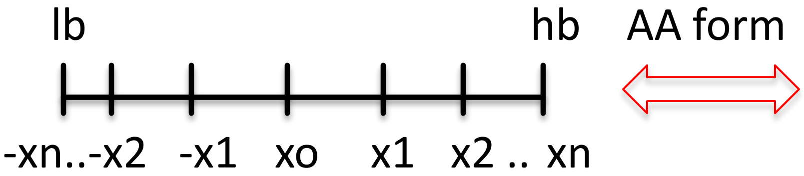


Agenda

- Overview of state of art approaches
- Related work: Affine Arithmetic (AA)
- Extended Affine Arithmetic for Systems with Discontinuities
- Benchmark: 3rd Order Sigma-Delta Modulator
- Conclusion

Related work: Range Arithmetics, AA

- Compute a system behavior for the set of variation/uncertain values enclosed within ranges
- Ranges modeled using Affine Arithmetic (AA)[Andrade '94]



$$\tilde{x} = x_0 + \sum_{i=1}^{n} \varepsilon_i x_i \quad \varepsilon_i \in [-1, 1]$$

$$lb = x_0 - rad(\tilde{x}) = x_0 - \sum_{i=1}^{n} |x_i|$$

$$hb = x_0 + rad(\tilde{x}) = x_0 + \sum_{i=1}^{n} |x_i|$$

- AA Properties
  - Handles the dependency problem in Interval Arithmetic $\tilde{x} - \tilde{x} = 0$
  - Exact computation result for affine operations
  - Results of non-affine operations over approximated

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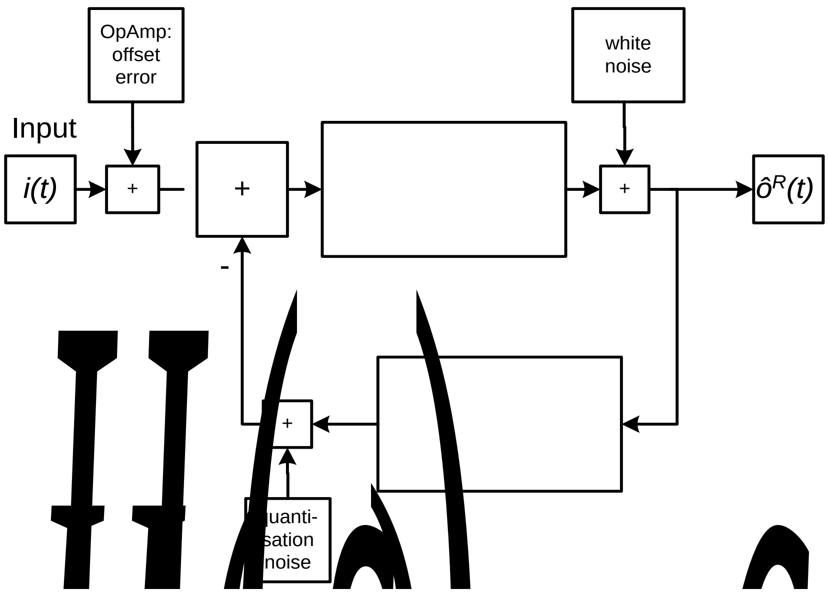
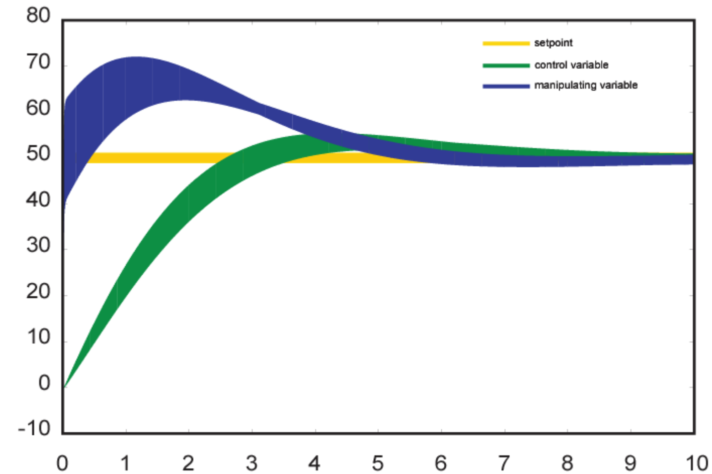
Control loop

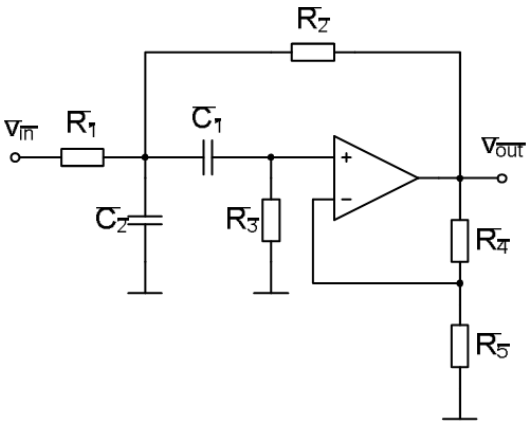
C. Grimm, W. Heupke, K. Waldschmidt: „Refinement of Mixed-Signal Systems with Affine Arithmetic“, DATE ‘04, 2004.

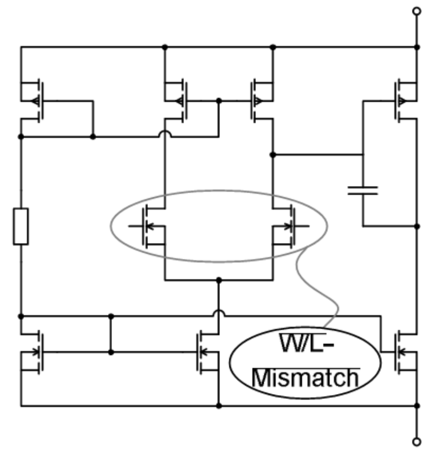




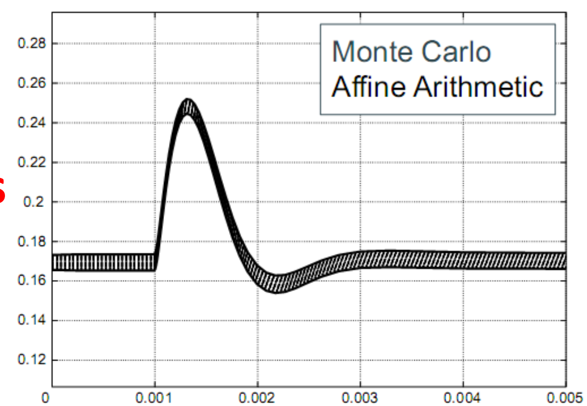
Analog circuits

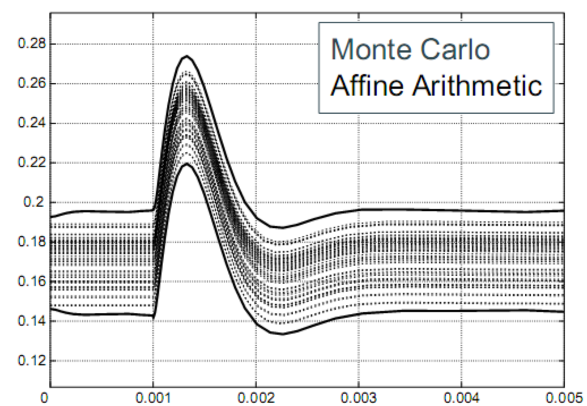
D. Grabowski, C. Grimm, E. Barke: „Symbolic Modeling and Simulation of Circuits and Systems“, ISCAS ‘06, 2006.





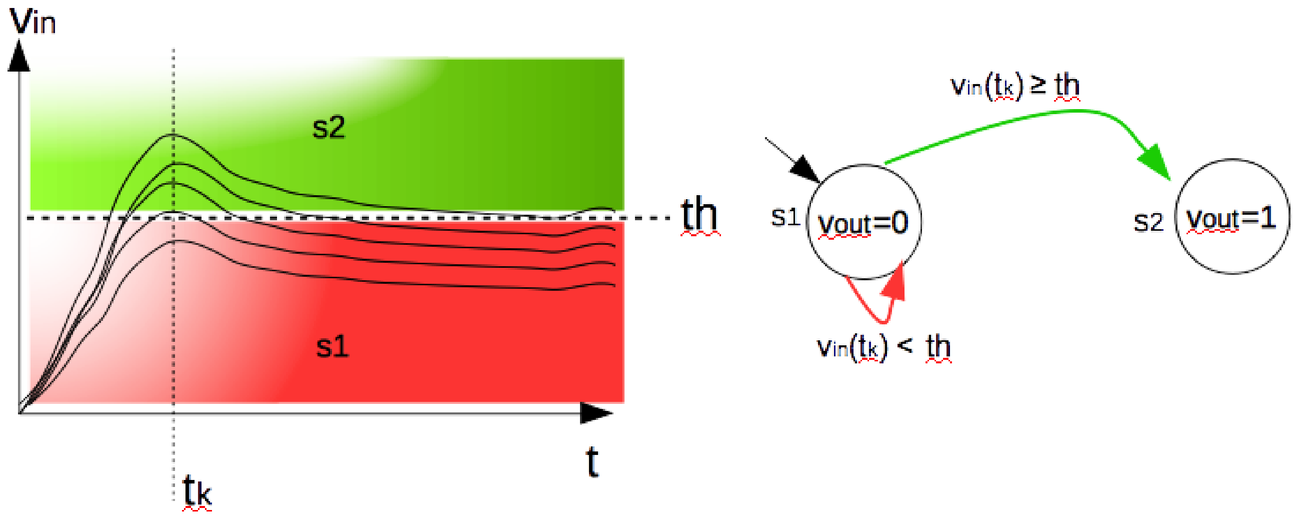
AAF:
5 sec.
50 runs (M.C.):
50 sec.





Limitations of AA: Discontinuities

- Modeling with AA limited to the analog continuous domain
- No models for digital components as comparators, quantizers, PLL phase/frequency detectors, ADCs, etc.



- Requires extension to handle discontinuities in a MS system

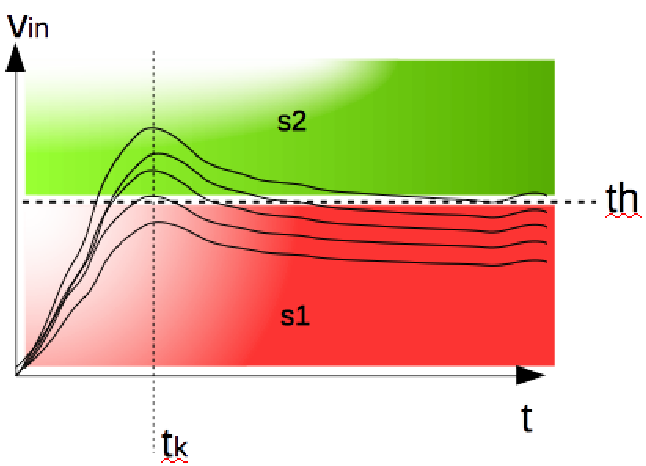
Extended Affine Arithmetic (X-AAF)

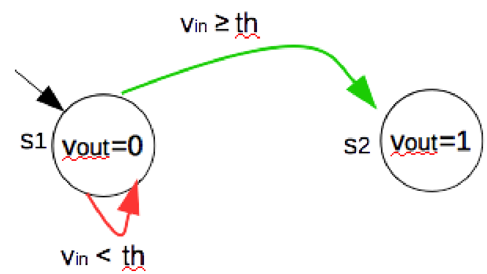
- System discontinuities are handled with deviation symbols
  $\omega_i \in \{-1, 1\}, i \in \mathbb{N}$
- X-AAF is defined as:

$$\hat{y} = \tilde{y}_0 + \sum_{i \in \mathbb{N}} \omega_i \tilde{y}_i \quad \tilde{y}_0, \tilde{y}_i \in AAF$$

$\tilde{y}_0$ - mean value

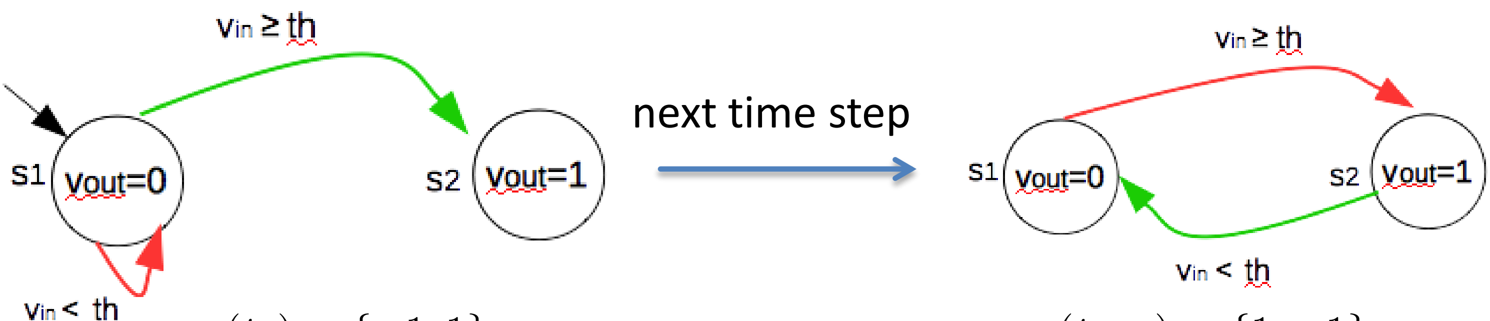
$\tilde{y}_i$ - deviation from the mean value





$$V_{out} = \{0, 1\} = 0.5 + \omega_1 0.5$$

Why {-1, 1} for ω?



$$\omega(t_k) = \{-1, 1\}$$

$$v_{out}(t_k) = \{0, 1\} = 0.5 + \omega(t_k)0.5$$

$$\omega(t_{k+1}) = \{1, -1\} = -\omega(t_k)$$

$$v_{out}(t_{k+1}) = 0.5 - \omega(t_k)0.5$$

The X-AAF implementation

- X-AAF is implemented as Abstract Data Type
- The same concept can be implemented in any simulator which supports the use of ADT (by replacing double/int with XAAF)
- We did this for SystemC AMS
- In SystemC AMS signals are instantiated with

  sca_tdf::sca_signal<T> some_signal;

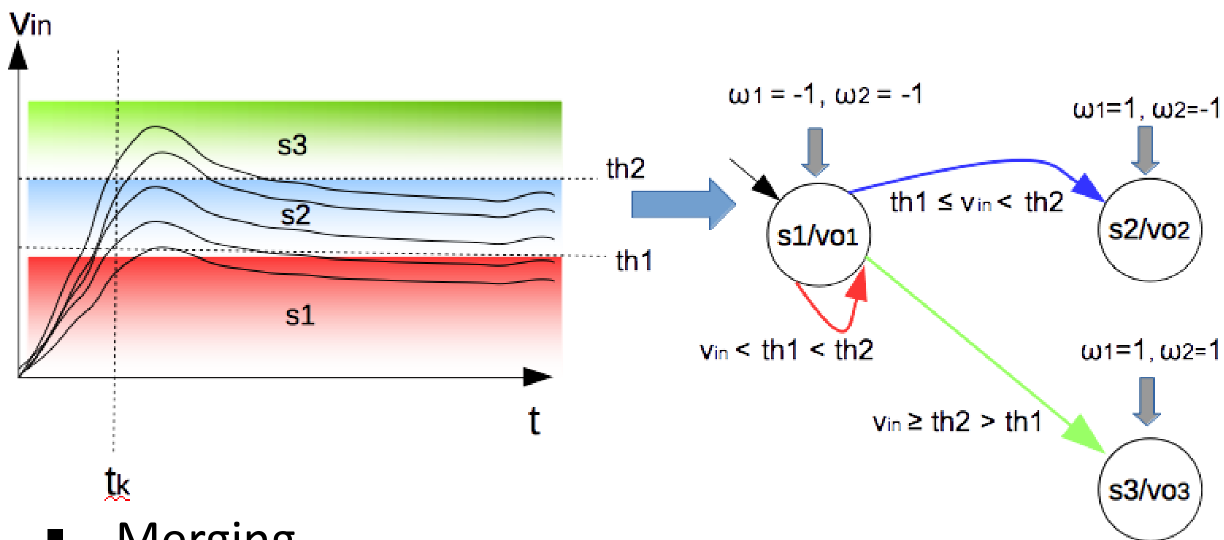
  T- template parameter specifying the type of a signal value
- Example:

  A signal whose value is real number is instantiated with

  sca_tdf::sca_signal<double> some_signal;
- Signal with XAAF type value:

  sca_tdf::sca_signal<XAAF> some_signal;

Split and Merge operations for Control Flow Graphs



$$\omega_2 = \begin{cases} 1 : vin \geq th2 \\ -1 : vin < th2 \end{cases}$$

$$\omega_1 = \begin{cases} 1 : vin \geq th1 \\ -1 : vin < th1 \end{cases}$$

- Merging

$$\{v_{01}, v_{02}, v_{03}\} = \{v_{01}, v_{02}\} \oplus \{0, v_{03} - v_{02}\}$$

$$\frac{v_{01}+v_{02}}{2} + \omega_1 \frac{v_{02}-v_{01}}{2} + \frac{v_{03}-v_{02}}{2} + \omega_2 \frac{v_{03}-v_{02}}{2}$$

$$\frac{v_{01}+v_{02}}{2} + \frac{v_{03}-v_{02}}{2} + \omega_1 \frac{v_{02}-v_{01}}{2} + \omega_2 \frac{v_{03}-v_{02}}{2} \qquad \omega_1, \omega_2 \in \{-1, 1\}$$

Computation with XAAF terms

- Computation with XAAF terms requires operator overloading
- Overloaded binary arithmetic operators:
  - Addition and subtraction operator +, +=, -, -=

$$\hat{x} \pm \hat{y} = \tilde{x}_0 \pm \tilde{y}_0 + \sum_{i=1}^{n} \omega_i (\tilde{x}_i \pm \tilde{y}_i)$$

  - Multiplication operator *, *=

1. Multiplication with constant

$$c\hat{x} = c\tilde{x}_0 + \sum_{i=1}^{n} \omega_i c\tilde{x}_i$$

2. Multiplication of two XAAF terms

$$\hat{x}\hat{y} = \tilde{x}_0\tilde{y}_0 + \sum_{i=1}^{n} (\tilde{x}_0\tilde{y}_i + \tilde{x}_i\tilde{y}_0)\omega_i + \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{x}_i\tilde{y}_j\omega_i\omega_j$$

$$\omega_i\omega_j = \begin{cases} 1 & : i = j \\ \omega_{n+k} & : i \neq j \end{cases}$$

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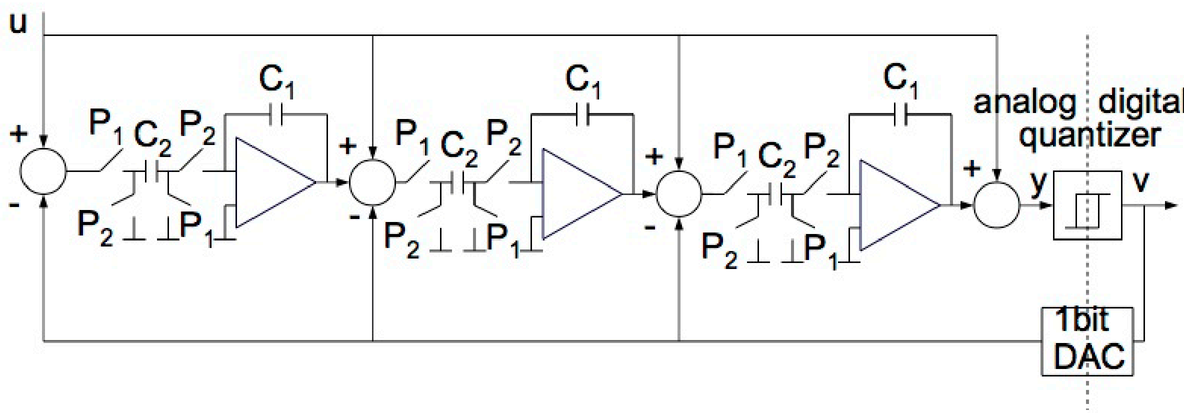
Benchmark: 3rd Order Sigma-Delta Modulator



$$\frac{C_2^{(1)}}{C_1^{(1)}} = \frac{0.07333(1+\varepsilon_1 0.3+\varepsilon_2 0.05)}{(1+\varepsilon_1 0.3+\varepsilon_3 0.05)}$$

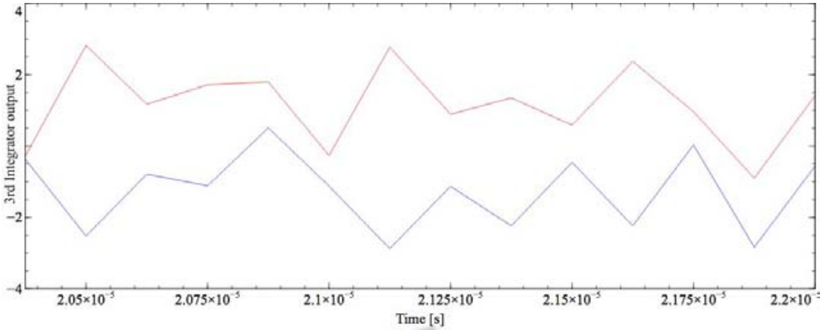
$$\frac{C_2^{(2)}}{C_1^{(2)}} = \frac{0.2881(1+\varepsilon_1 0.3+\varepsilon_2 0.05)}{(1+\varepsilon_1 0.3+\varepsilon_3 0.05)}$$

$$\frac{C_2^{(3)}}{C_1^{(3)}} = \frac{0.7997(1+\varepsilon_1 0.3+\varepsilon_2 0.05)}{(1+\varepsilon_1 0.3+\varepsilon_3 0.05)}$$

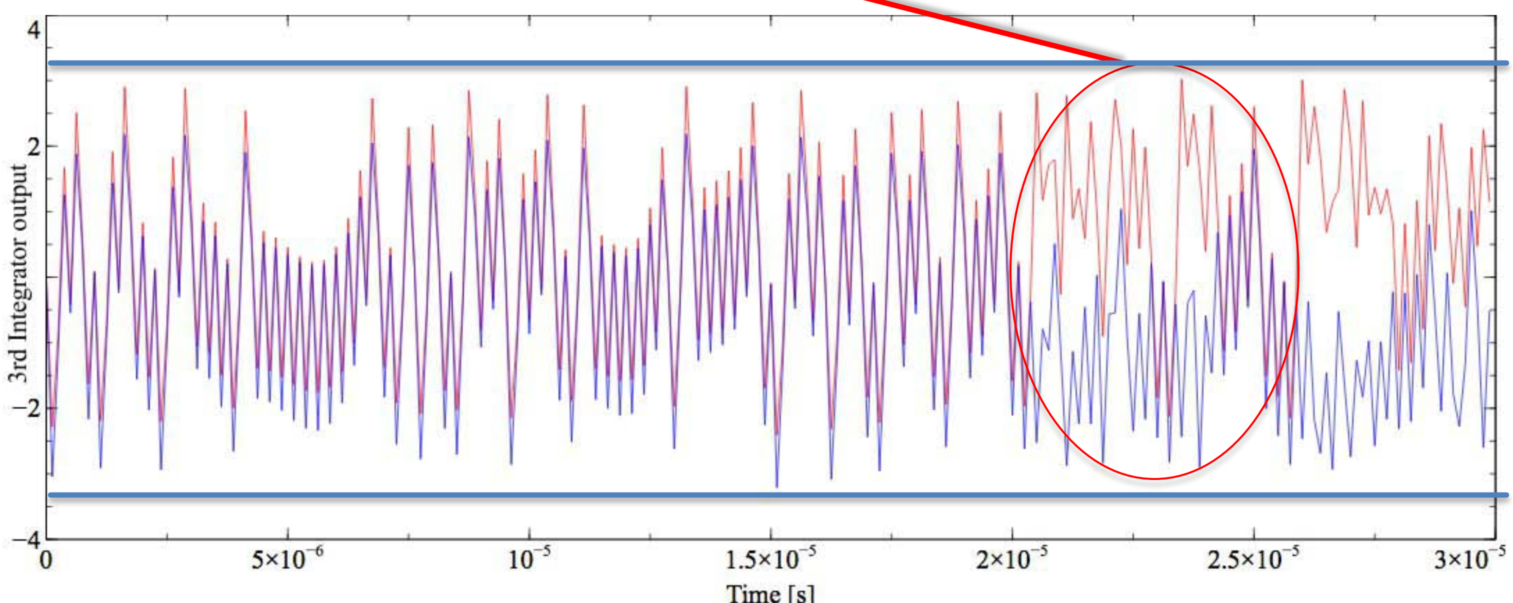
$$u(t) = 0.6sin(2\pi * 3.9 * 10^3 * t)$$

$$f_s = 8MHz$$

Integrator outputs




```
2.05e-05 -------------
Length of XAAF = 1
mean_value =
-------------
Length = 4
v0     = -0.197409
Radius = 0.0280581
e1 -> 0
e2 -> -0.00977025
e3 -> 0.00977025
e4 -> -0.00851765
-------------
w1 ->
-------------
Length = 4
v0     = -2.66608
Radius = 0.378935
e1 -> -0
e2 -> -0.13195
e3 -> 0.13195
e4 -> -0.115034
-------------
```



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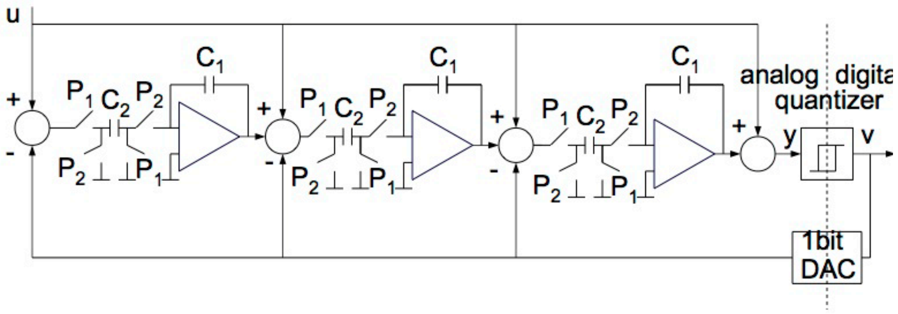
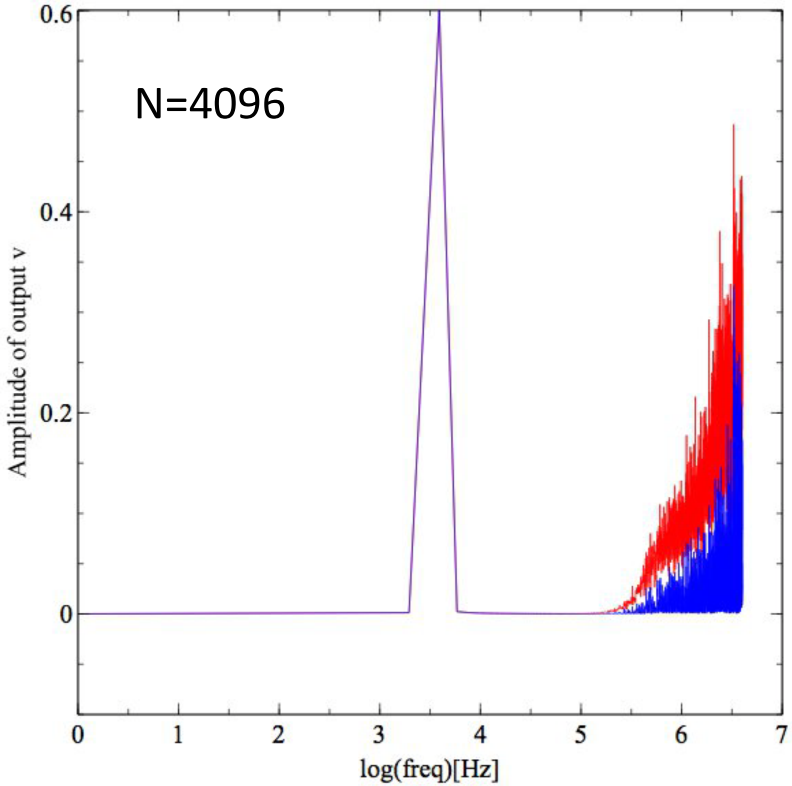
FFT of the modulator output v



Simulation time + FFT computation took 23 minutes.

The number of ω symbols used to cover all possible cases was 236 << 4096



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Conclusion

- Easy integration of the approach into existing design flows and simulators
- Scalability (Mostly linear)
- Simulation of MS systems for the set of input conditions, variations….
  (Problem coverage increased)
- Symbolic representation of system response allows efficient sensitivity analysis

Thank you for your attention!