Predictive Global Optimization with Response Surface Modeling

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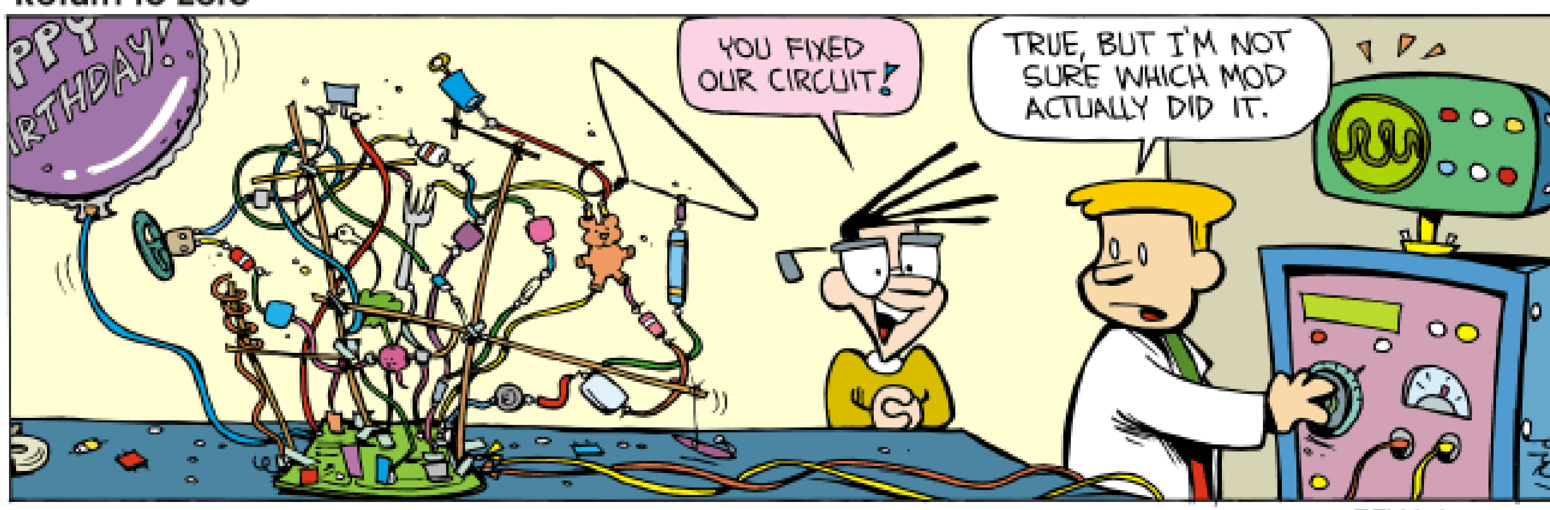
Contents

- Introduction and motivation
- Overall algorithm
- Details on algorithm components
- LC-VCO optimization example

Schematic Level Circuit Design

- It's an optimization problem
  - Many design variables to consider
  - Also many performance constraints
  - Optimization needed for some performance(s)

Return to Zero



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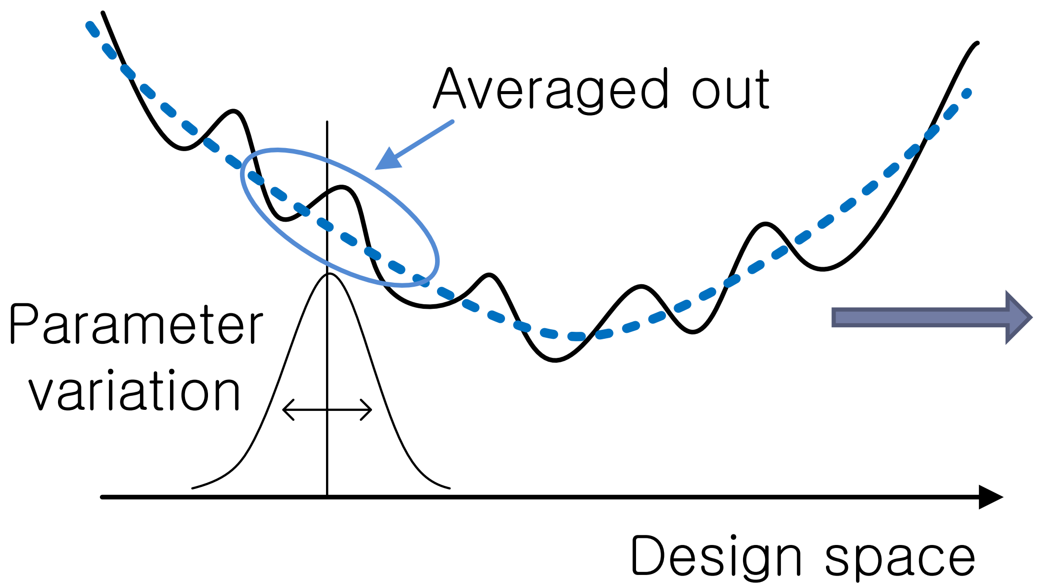
Previous Approaches

- Equation-based ones
  - convex optimization techniques
  - Fast execution
  - Accuracy and labor issues

- Simulation-based ones
  - Response surface modeling(RSM)
  - Genetic algorithms
  - SPICE level accuracy

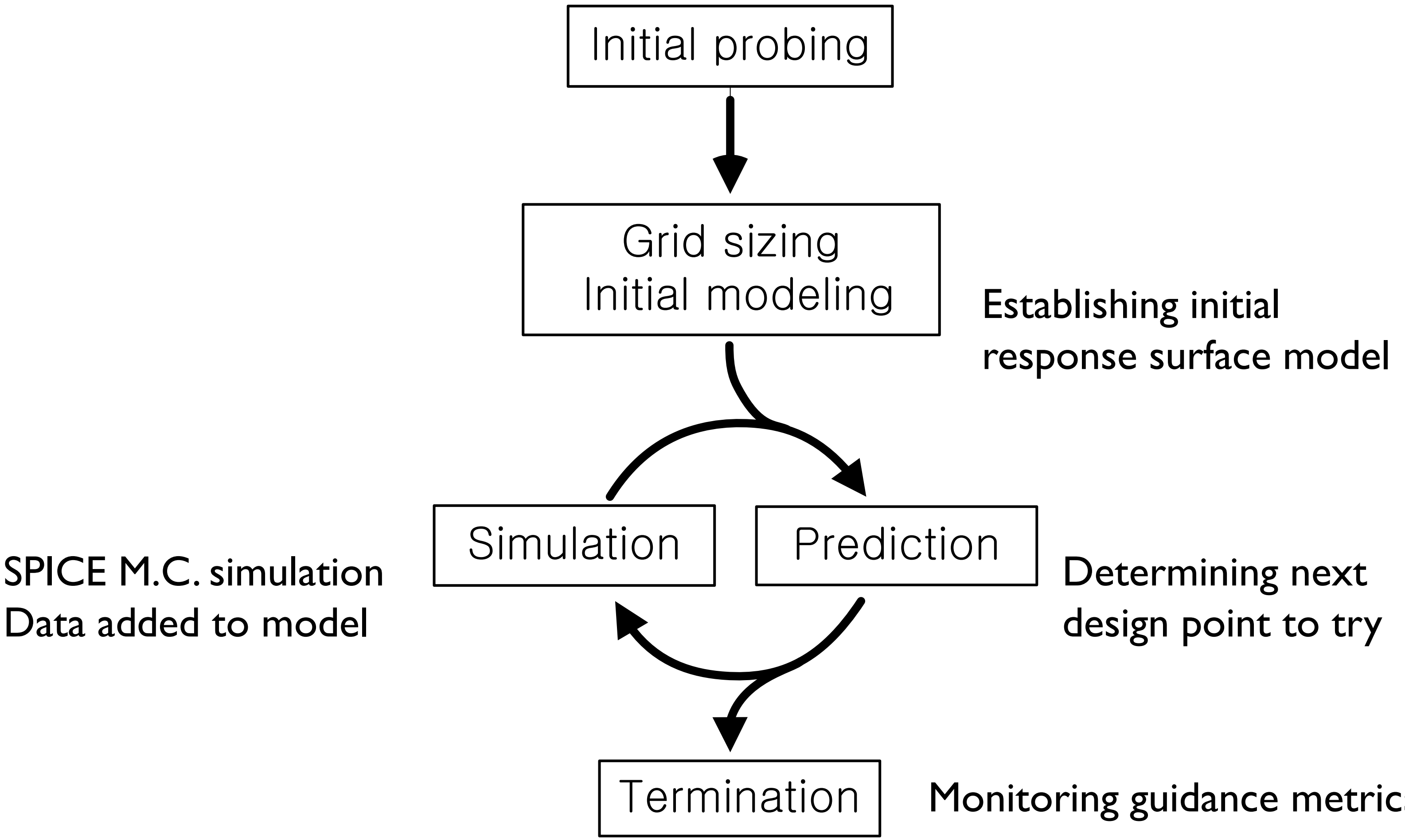
Motivation for RSM Method

- Circuit performance function is smooth
  - Design parameter variation averages out surface roughness
  - Correlation models the amount of smoothness



Interpolation with discrete design points is enough

Overall Algorithm



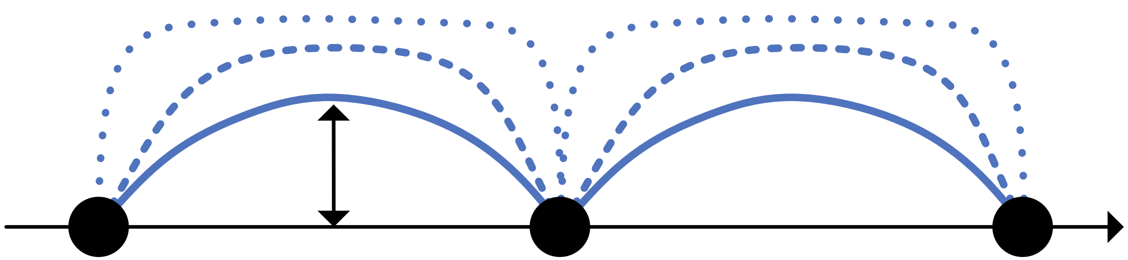
Introducing Discrete Grid

- Using correlation of target performance function
  - Gaussian function for modeling correlation
    - e.g) Two points should have at least 0.5 correlation

$$\rho(\Delta_d) = e^{-\|\theta\ \Delta_d\|^2} \geq 0.5$$

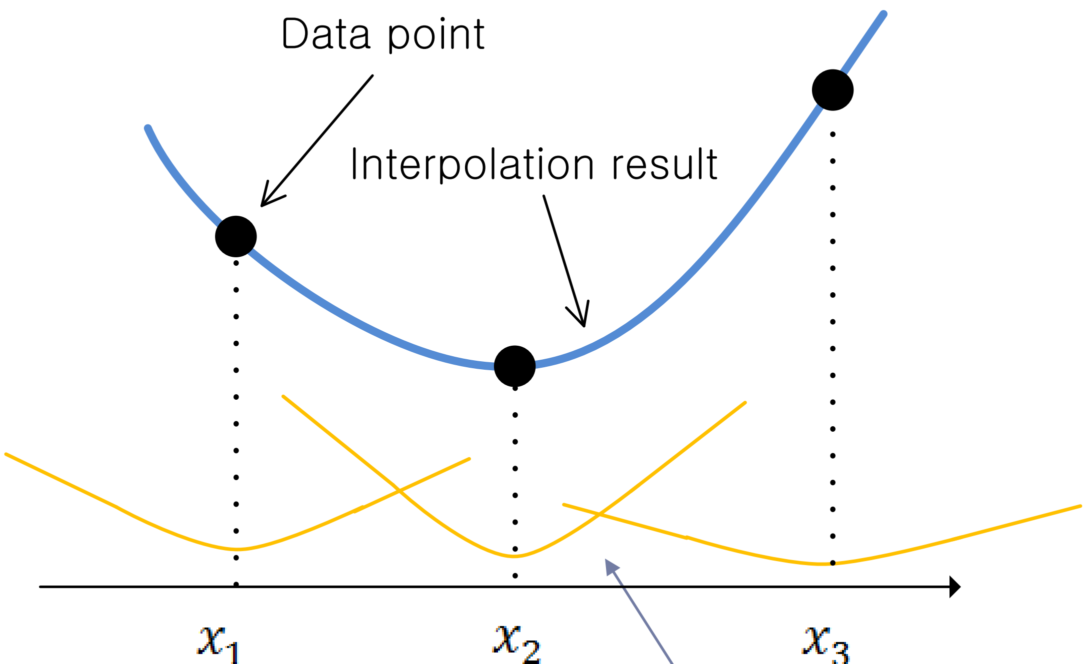
$$\Delta_d \leq \frac{1}{\theta} \log 2$$

- Also related to interpolation uncertainty
  - The amount of uncertainty is determined by correlation



Radial Basis Function Interpolation

- Linear combination of basis functions
- Multiquadric bases can do extrapolation



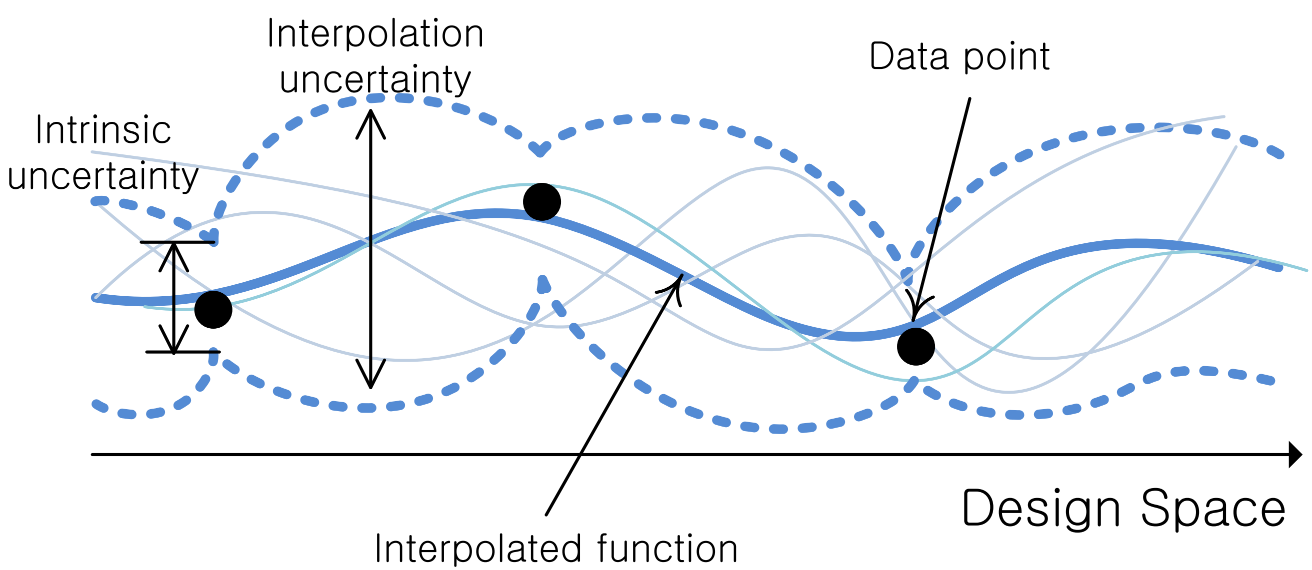
$$\hat{f}(x) = \sum_{i=1}^{n} \beta_i \phi(x, x_i)$$

$$\hat{f}(x_i) = f(x_i)$$

$$\phi(x, x_2) = \sqrt{|x - x_2|^2 + c^2}$$

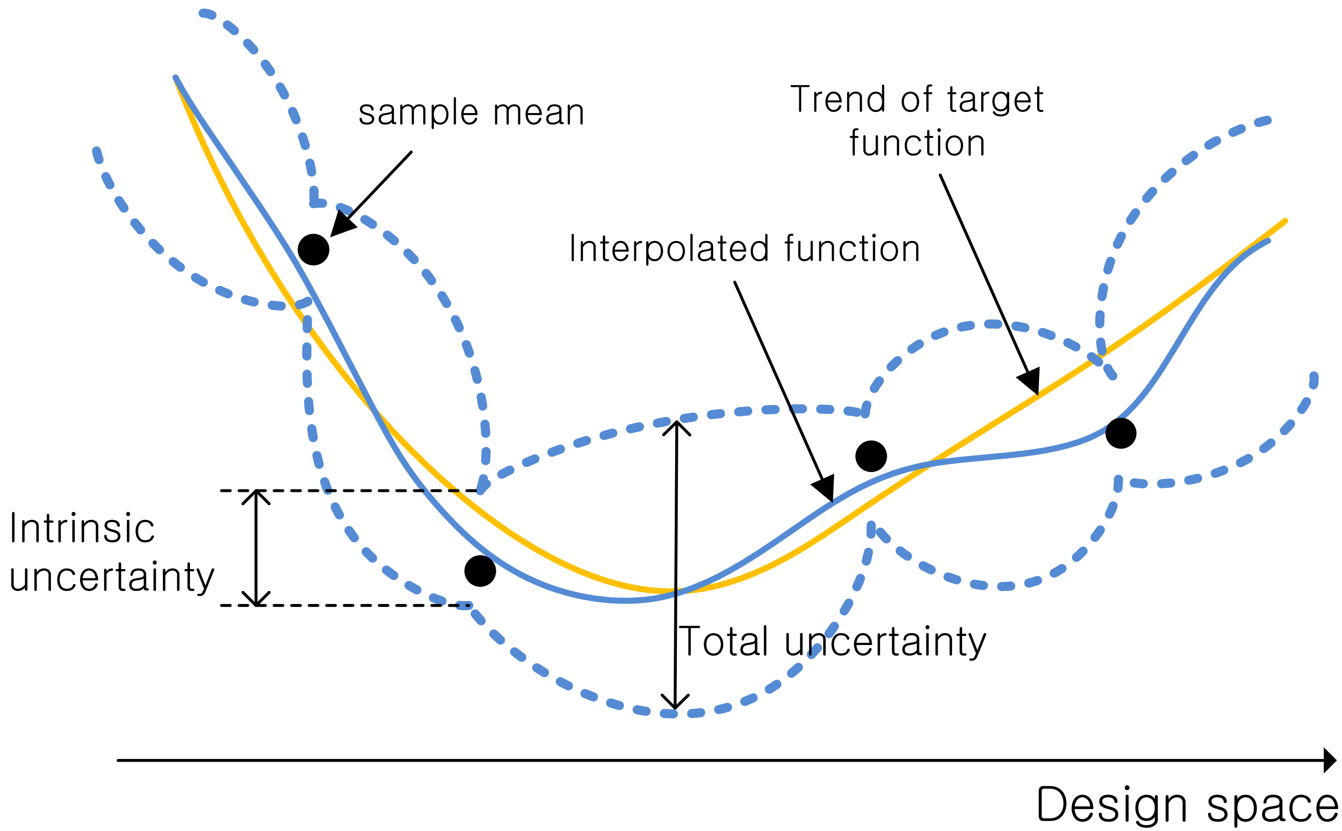
Stochastic Kriging

- Estimate intermediate values using correlation
  - Also gives regularization for noisy data
- Measure for interpolation uncertainty
  - Intrinsic uncertainty and extrinsic uncertainty



Desirable Regression Method

- Combining RBF interpolation and stochastic Kriging
  - Model-order reduction applied on RBF term



Model Equation

Multiquadric RBF's — Gaussian Process

$$\mathrm{Y}(\mathbf{d}) = \boldsymbol{\phi}^{T}(\mathbf{d})\boldsymbol{\beta} + Z(\mathbf{d}) + \mathrm{V}(\mathbf{d})$$

Mean surface — Noise

$$\boldsymbol{\phi}^{T}(\mathbf{d})\boldsymbol{\beta} = \boldsymbol{\Sigma}_{\mathbf{j}}\left(\phi\left(\boldsymbol{d}, \boldsymbol{d}_{j}\right)\boldsymbol{\beta}_{j}\right)$$

$$\phi(\mathrm{d}, \mathrm{d}') = \sqrt{\|\mathrm{d} - \mathrm{d}'\|^{2} + c^{2}}$$

Rough global trend

$$Z(\mathbf{d}) \sim GP(0, \boldsymbol{\Sigma}_{\mathbf{Z}})$$

$$\Sigma_{\mathrm{Z}}(\mathrm{d}, \mathrm{d}') = \sigma_{\mathrm{Z}}^{2}\, e^{-\left\|\theta^{\mathrm{T}}(\mathrm{d}-\mathrm{d}')\right\|^{2}}$$

Local deviation

$$V(\mathbf{d}) \sim GP(0, \boldsymbol{\Sigma}_{\mathbf{V}})$$

$$\Sigma_{\mathrm{V}}(\mathrm{d}, \mathrm{d}') = \sigma_{\mathrm{V}}^{2}(\mathrm{d})\, \delta(\mathrm{d}, \mathrm{d}')$$

Noise in data

Regression Equations

- Linear unbiased minimum mean-squared-error

Interpolated Value

$$\hat{Y}(d_{new}) = \underbrace{\boldsymbol{\phi}^{\mathrm{T}}(\mathbf{d}_{new})\hat{\boldsymbol{\beta}}}_{\text{Extrapolative trend}} + \underbrace{\mathbf{r}^{\mathrm{T}}(\mathbf{d}_{new})\mathbf{K}^{-1}\overbrace{\left(\overline{\mathrm{Y}}(\boldsymbol{D}) - \boldsymbol{\Phi}\hat{\boldsymbol{\beta}}\right)}^{\text{Deviation from trend}}}_{\text{Compensation}}$$

Total uncertainty

$$MSE(d_{new}) = \underbrace{\sigma_V^2(d_{new})}_{\text{Intrinsic variation}} + \underbrace{\sigma_Z^2\left(1 - [\boldsymbol{\phi}^T \quad \mathbf{r}^T]\begin{bmatrix} 0 & \boldsymbol{\Phi}^T \\ \boldsymbol{\Phi} & \mathbf{K} \end{bmatrix}^{-1}\begin{bmatrix} \boldsymbol{\phi} \\ \mathbf{r} \end{bmatrix}\right)}_{\text{Interpolation uncertainty}}$$

Guidance Metrics for Optimization

- Determining next point to simulation
  - Approximate performance distribution on unvisited design point

$$\hat{m}(d_{new}) \sim N(m_0, \epsilon_m^2)$$
$$\hat{s}(d_{new}) \sim N(s_0, \epsilon_s^2)$$
$$\rightarrow f(d_{new}) \sim N(\hat{m}(d_{new}), \hat{s}(d_{new})^2)$$

  - For constraints, probability for feasibility

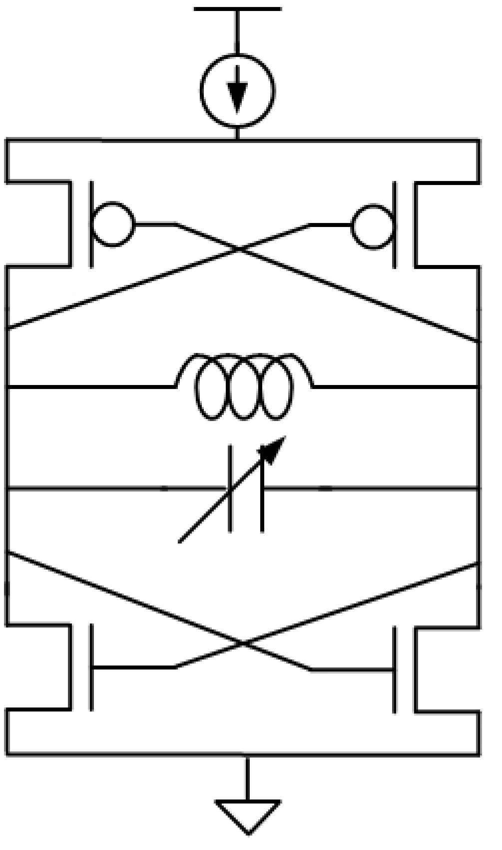
$$\mathrm{Prob}(\,f(d_{new}) > \theta\,)$$

  - For objective, expected improvement

$$E[\max(0, f(d_{new}) - f^*)]$$

LCVCO Optimization

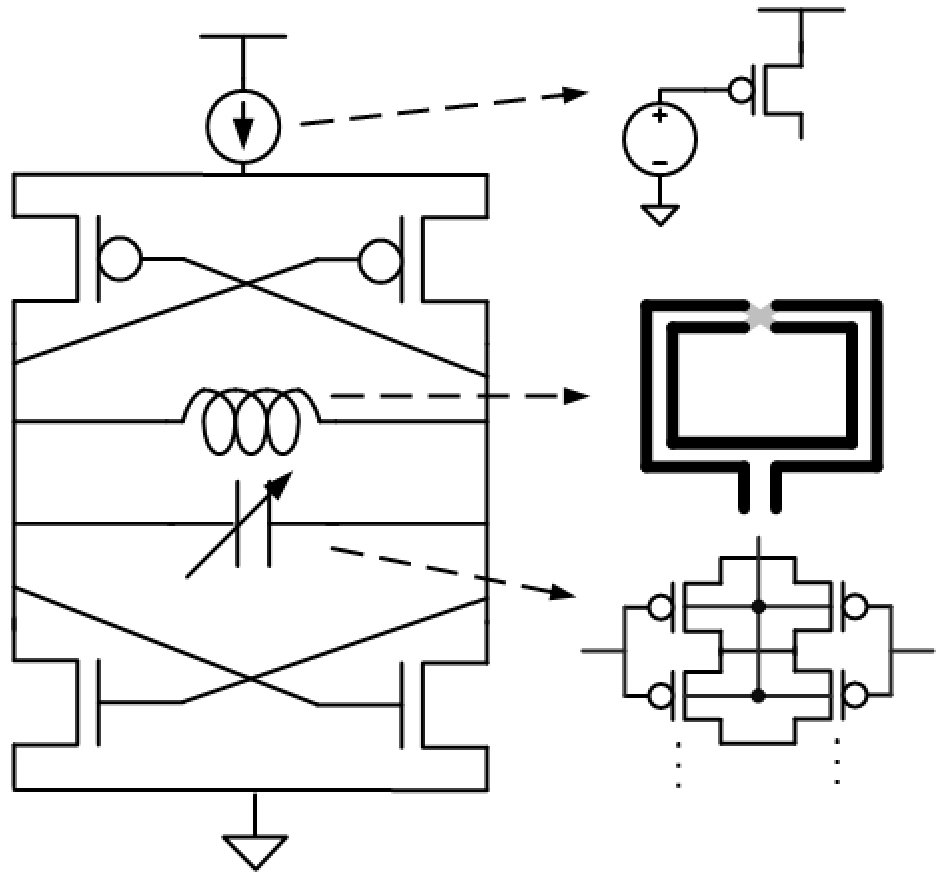
- Frequency Range
  - Min frequency < 6GHz
  - Max frequency > 7GHz
- Startup condition
  - $\frac{g_m^{-1}}{R_{tank}} > 5$
- Phase noise
  - Minimize @ 10MHz offset



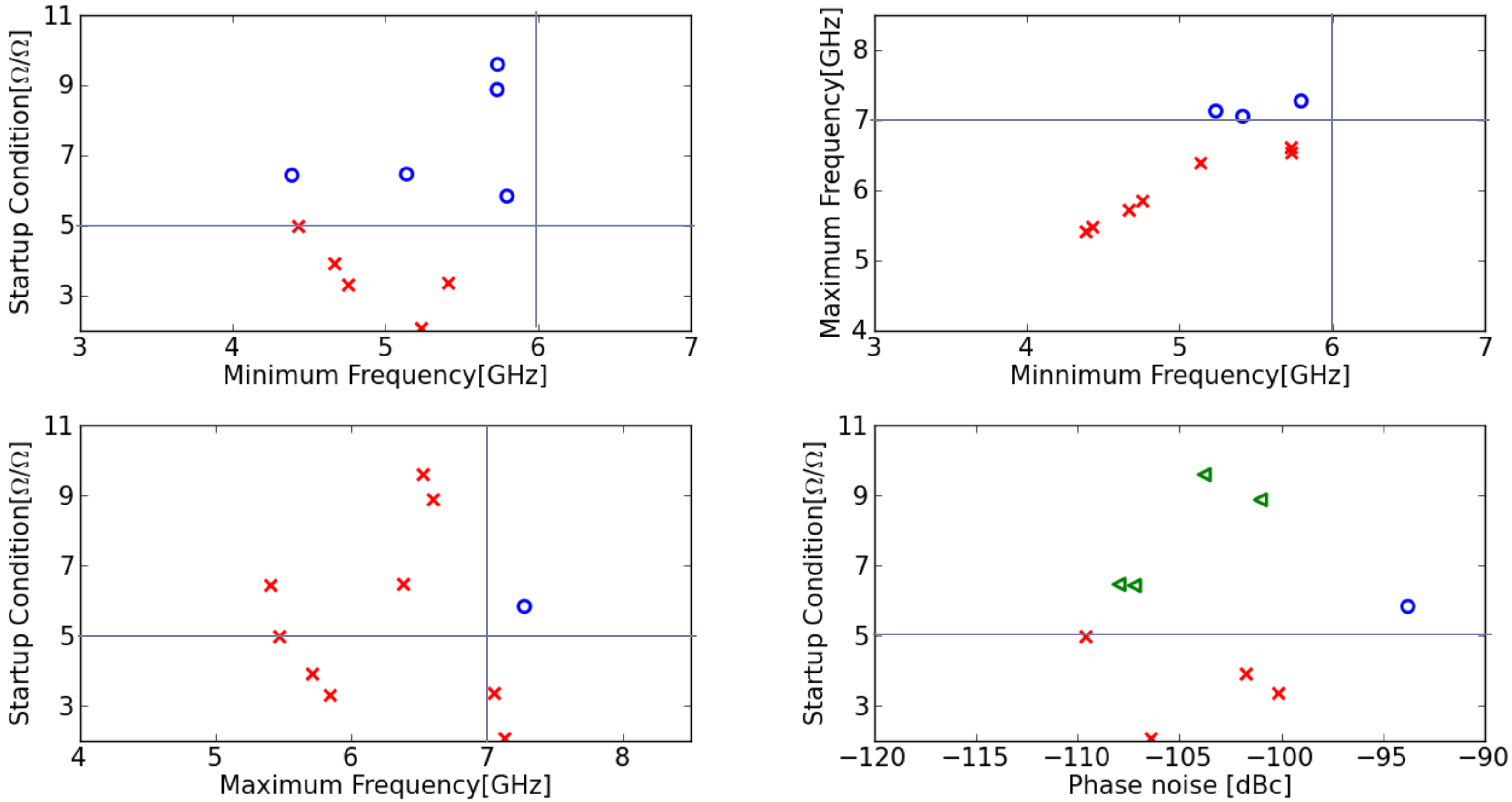
* Fixed power consumption

LCVCO Optimization

- Design variables
  - Gm cells
    - PMOS widths
    - NMOs widths
  - MOS Varactor
    - Number of MOS's
  - Spiral inductor
    - Radius
    - Track width

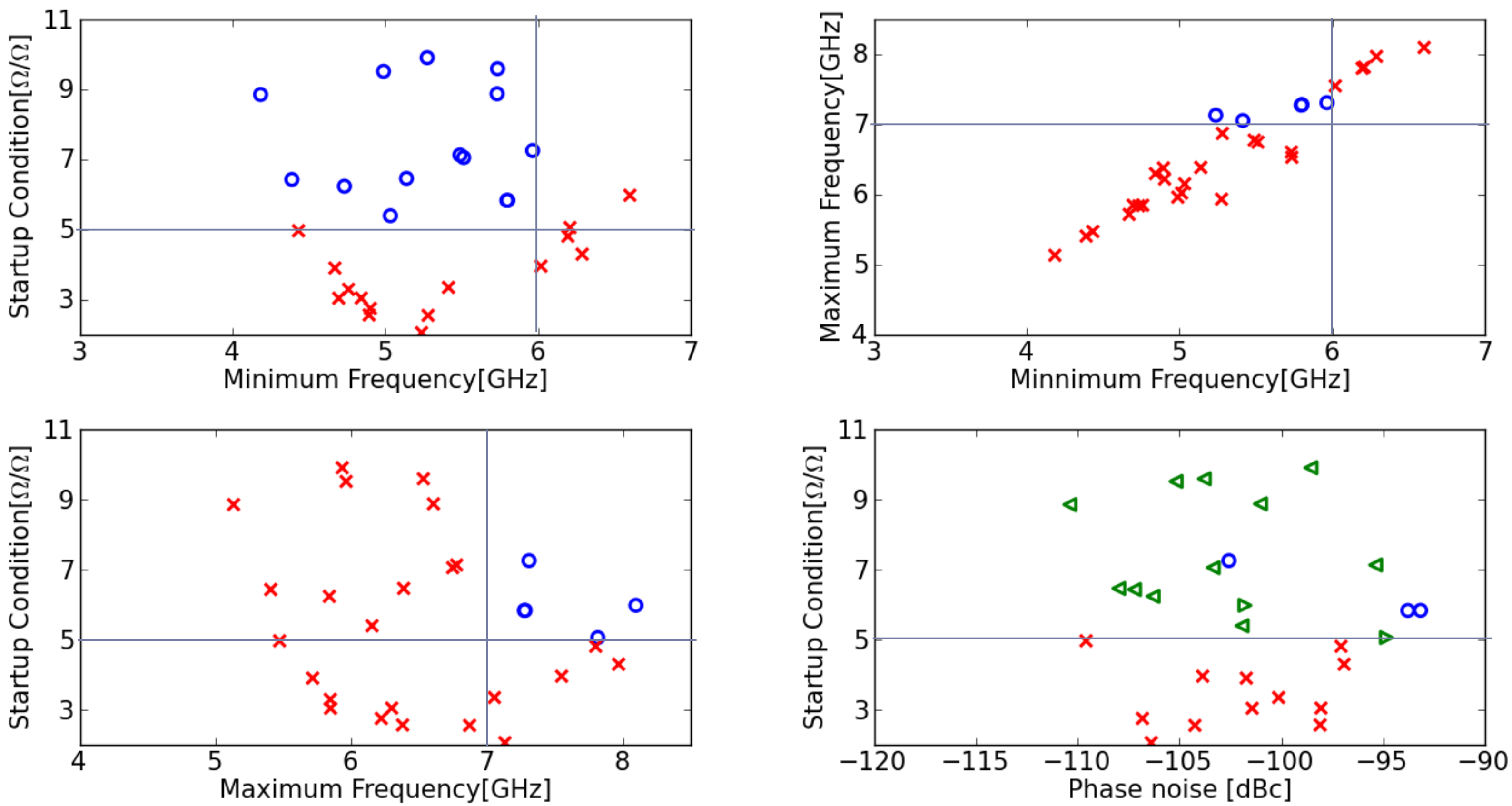


Performance Space Visualization



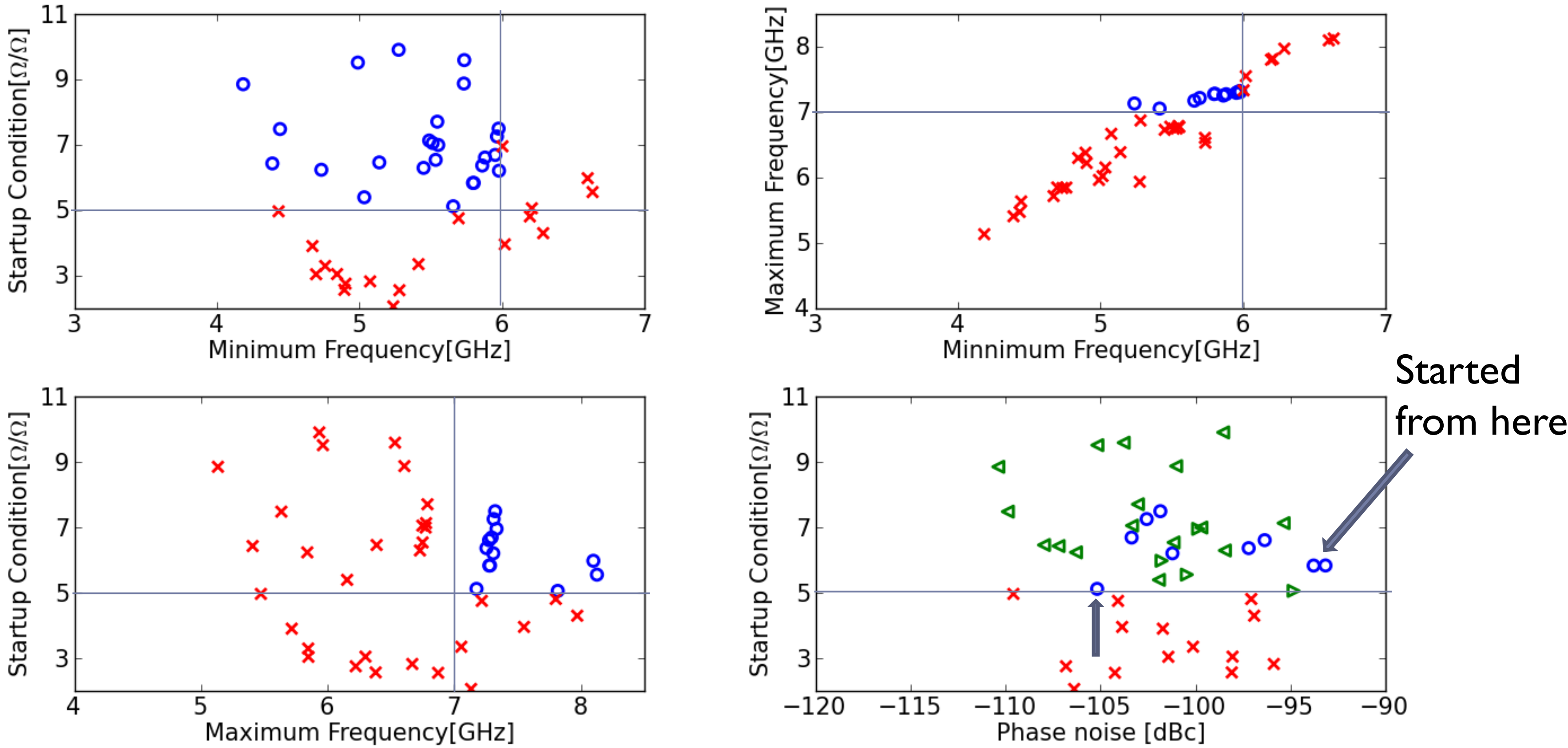
After initial random probing

Performance Space Visualization



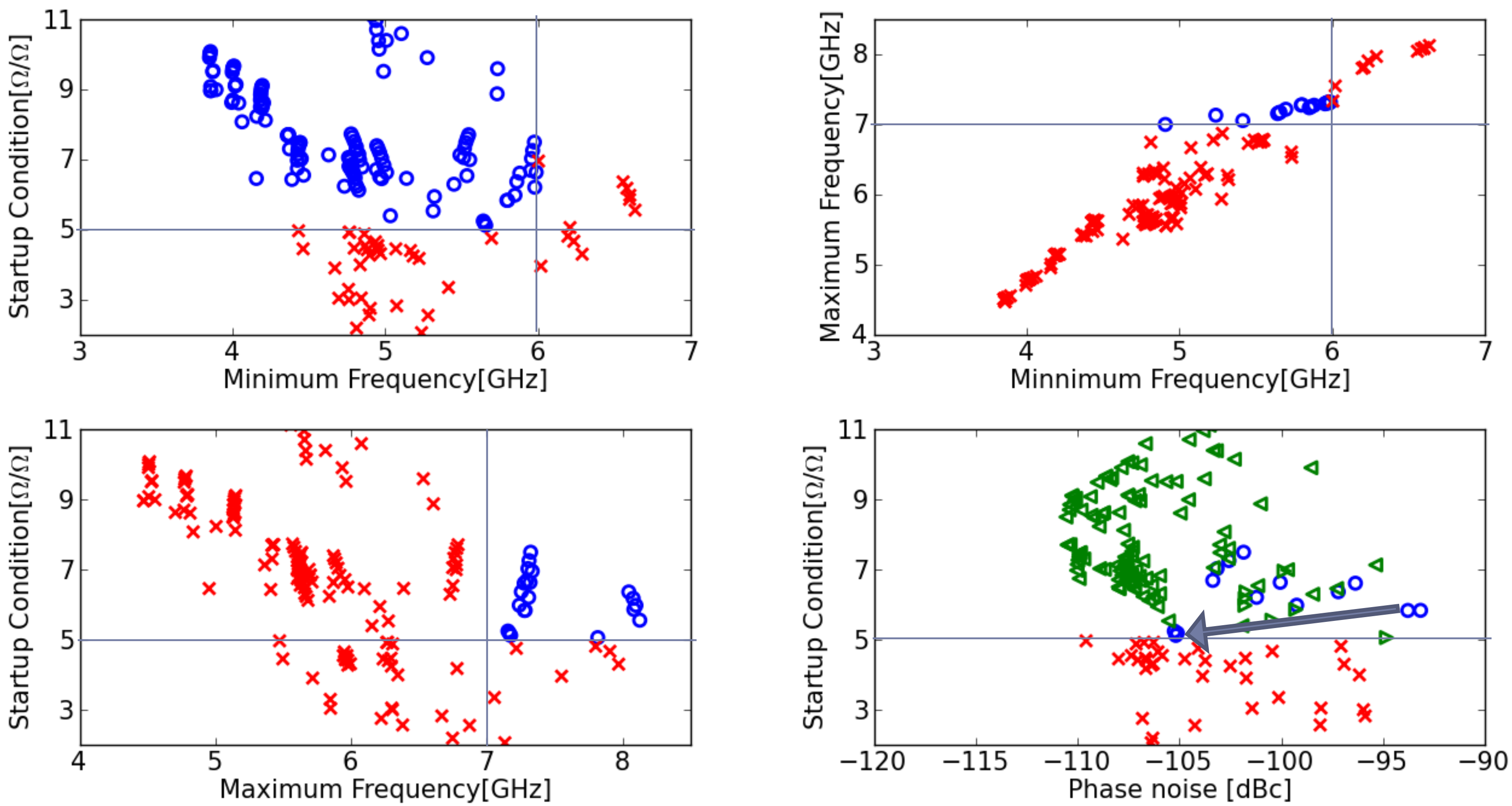
Model refinement for constraint metrics

Performance Space Visualization



Detection of optimum

Performance Space Visualization



Visiting additional data points with low probability for feasibility

Summary & Future work

- Global optimization with discretized design space
  - RBF and Stochastic Kriging are combined for modeling
  - Design space discretization based on correlation
- LCVCO circuit is tested for optimization
  - Early detection of optimum out of total iteration
- Extend to verification problems

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THANK YOU