Reachability Analysis Using Octagons

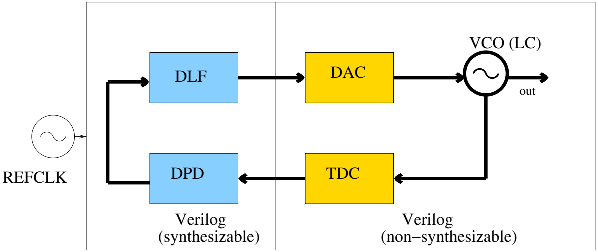
Andrew N. Fisher and Chris J. Myers

Department of Electrical and Computer Engineering
University of Utah

FAC 2014
July 9, 2014

Digitally Intensive Analog Circuits

- *Digitally intensive analog circuits* attempt to replace analog components with digital ones whenever possible.



- Result is optimized power efficiency and performance as well as improved robustness to process variability.
- These circuits though further complicate the verification problem.

Simulation-Based Verification

- Digital verification typically uses switch or RTL-level simulations.
- AMS verification uses detailed transistor-level (SPICE) simulations.
- SPICE simulation of a PLL can take weeks or even months.
- Long simulation time makes system-level simulation difficult.
- Functional bugs can be missed resulting in catastrophic failures.

Analog Verification

Sandipan Bhanot
CEO of Knowlent

If the digital designers did verification the way analog designers do verification, no chip would ever tape out.
(DACezine, January 2008)

Model Checking

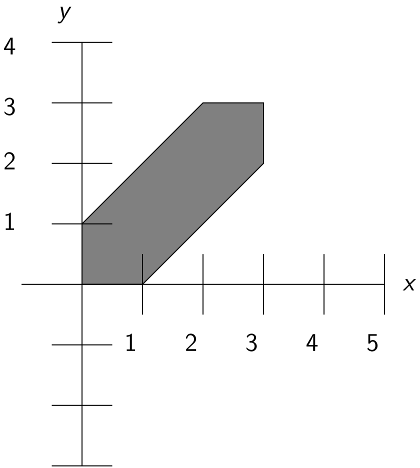
- *Model checking* uses non-determinism and state exploration to formally verify designs over all possible behaviors.
- Has had tremendous success for verifying of both digital hardware and software systems (now routinely used at Intel, IBM, Microsoft, etc.).
- For AMS circuits, it is a promising mechanism to validate designs in the face of noise and uncertain parameters and initial conditions.
- AMS verification is complicated by the need to:
  - Construct abstract formal models of the AMS circuits.
  - Specify formal properties that are to be verified.
  - Represent continuous variables efficiently (voltages, currents, and time).

Model Checking

- *Model checking* uses non-determinism and state exploration to formally verify designs over all possible behaviors.
- Has had tremendous success for verifying of both digital hardware and software systems (now routinely used at Intel, IBM, Microsoft, etc.).
- For AMS circuits, it is a promising mechanism to validate designs in the face of noise and uncertain parameters and initial conditions.
- AMS verification is complicated by the need to:
  - Construct abstract formal models of the AMS circuits. (FAC 2011)
  - Specify formal properties that are to be verified. (FAC 2013)
  - Represent continuous variables efficiently (voltages, currents, and time).

Zones

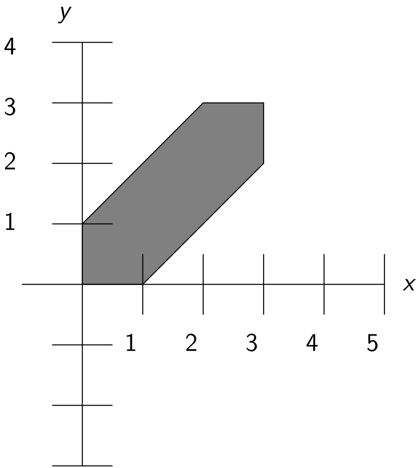
- Used for formal verification of timed automata and time(d) Petri nets.
- Simple geometric polyhedra formed by the intersection of hyper-planes representing inequalities of the form $y - x \leq c$.
- Implies polyhedra with only 0°, 90°, and positive 45° angles.
- For timed systems, all variables evolve at a rate of 1, and zone evolves along a positive 45° angle.
- Algorithms to restrict, project, and advance time are fast and simple.
- Can use Floyd's all pairs shortest-path algorithm to construct a canonical maximally tight representation.
- Conveniently represented using a *difference bound matrix* (DBM).



$$y - t0 \leq M_y$$
$$x - t0 \leq M_x$$
$$t0 - x \leq -m_x$$
$$t0 - y \leq -m_y$$
$$y - x \leq b_1$$
$$x - y \leq -b_2$$

$$\begin{array}{c} \\ t0 \\ x \\ y \end{array} \begin{array}{c} \begin{array}{ccc} t0 & x & y \end{array} \\ \begin{pmatrix} 0 & M_x & M_y \\ -m_x & 0 & b_1 \\ -m_y & -b_2 & 0 \end{pmatrix} \end{array}$$

Zones



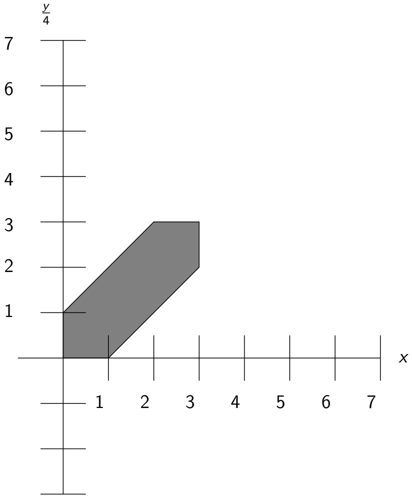
$$y - t0 \leq 3$$
$$x - t0 \leq 3$$
$$t0 - x \leq 0$$
$$t0 - y \leq 0$$
$$y - x \leq 1$$
$$x - y \leq 1$$

$$\begin{array}{c} \\ t0 \\ x \\ y \end{array}\begin{array}{c} \begin{array}{ccc} t0 & x & y \end{array} \\ \begin{pmatrix} 0 & 3 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

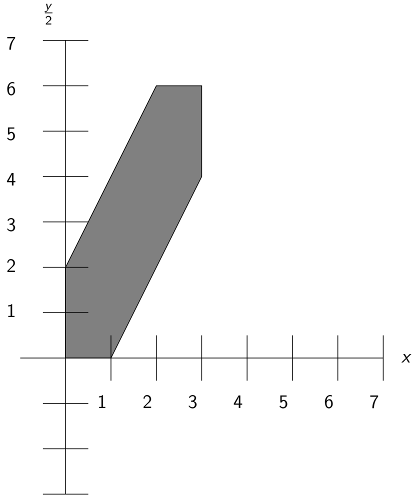
Zone Warping

- To verify AMS circuits, need variables that evolve at non-unity rates.
- Zones can be used with a variable substitution.
- Replace variable v with non-zero rate r with a variable $\frac{v}{r}$.
- The new variable $\frac{v}{r}$ evolves at a rate of 1.
- Resultant polyhedra is no longer a zone.
- *Warping* creates the smallest zone that contains it.

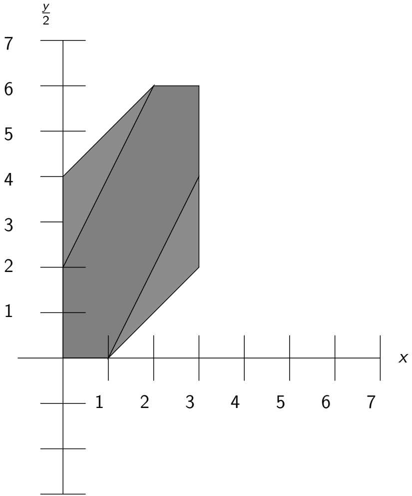
Positive Zone Warping



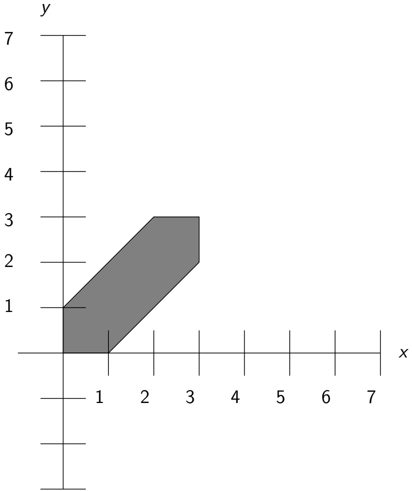
Positive Zone Warping



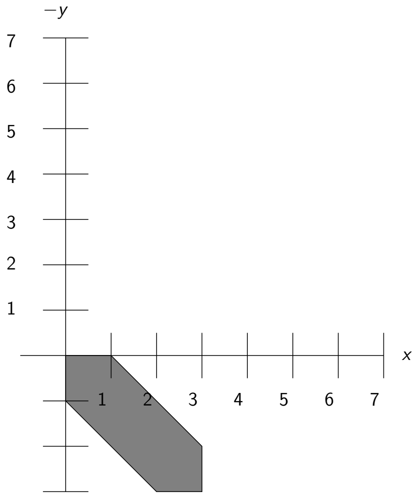
Positive Zone Warping



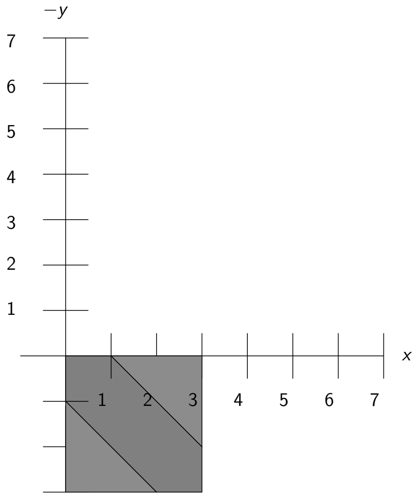
Negative Zone Warping



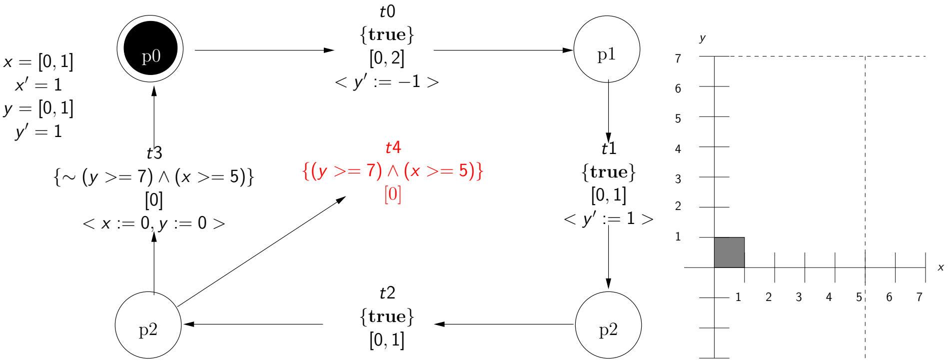
Negative Zone Warping



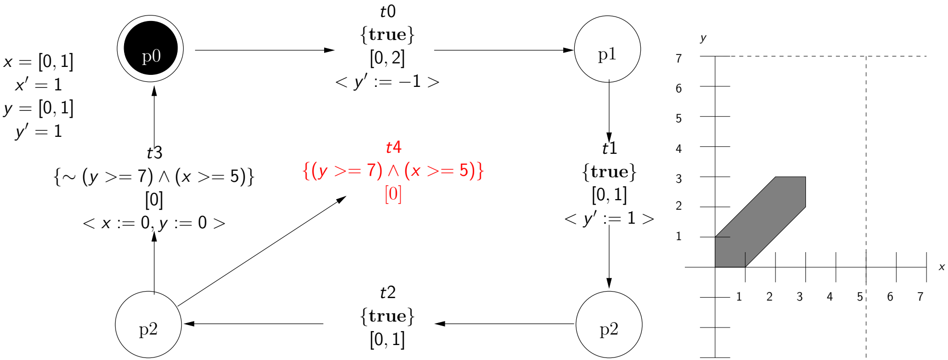
Negative Zone Warping



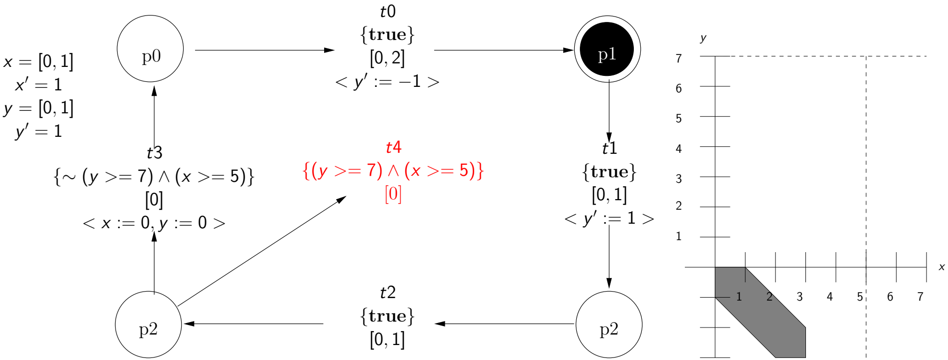
Negative Zone Warping: False Negative



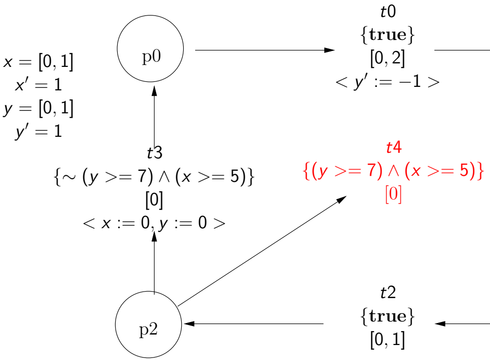
Negative Zone Warping: False Negative

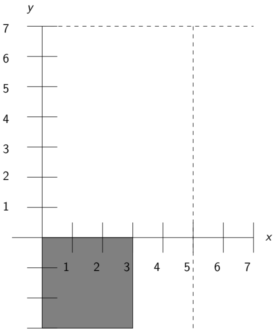


Negative Zone Warping: False Negative

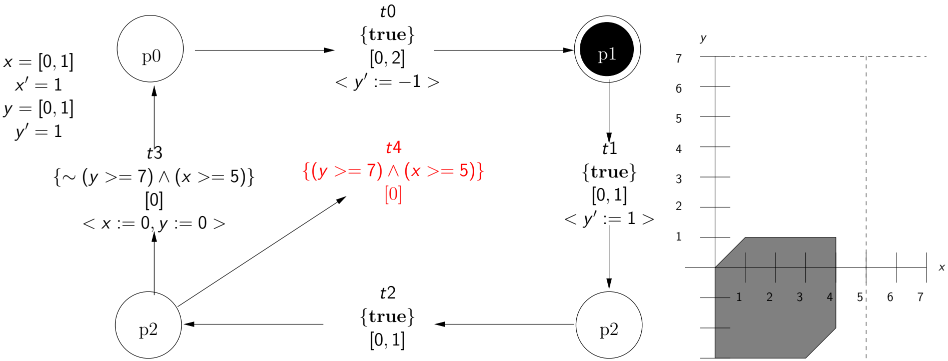


Negative Zone Warping: False Negative

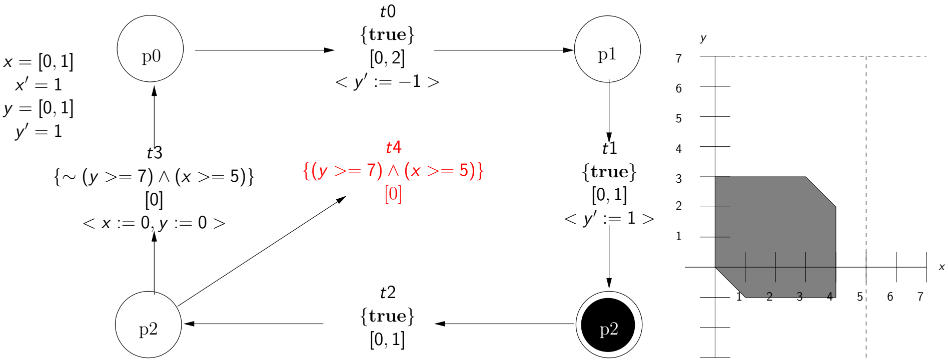




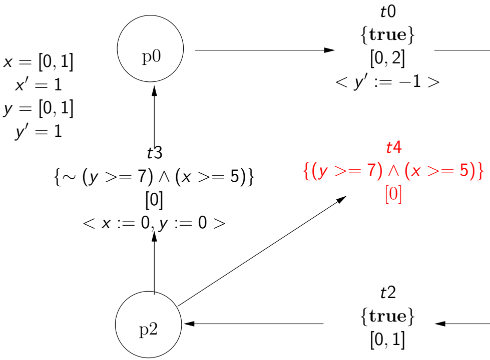
Negative Zone Warping: False Negative

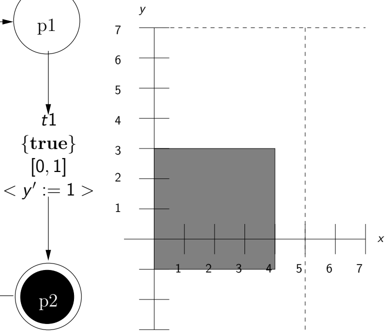


Negative Zone Warping: False Negative

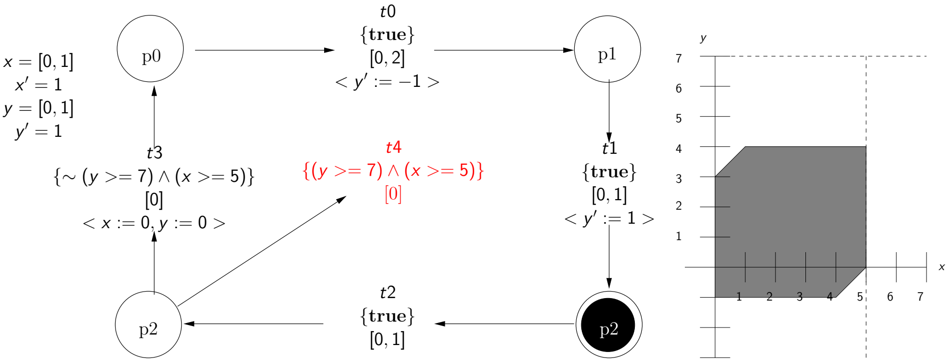


Negative Zone Warping: False Negative

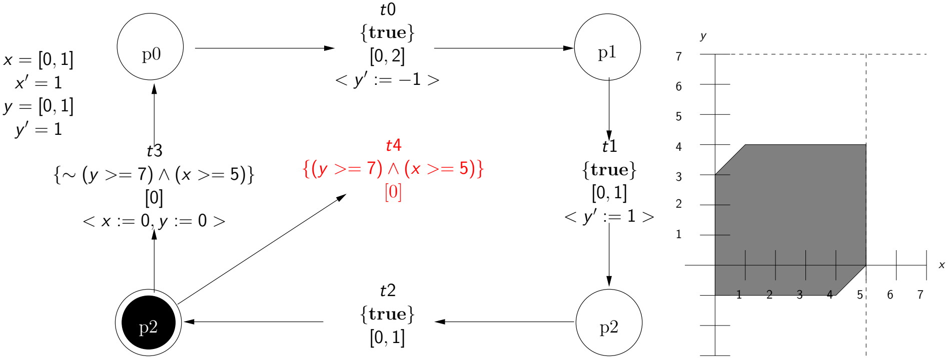




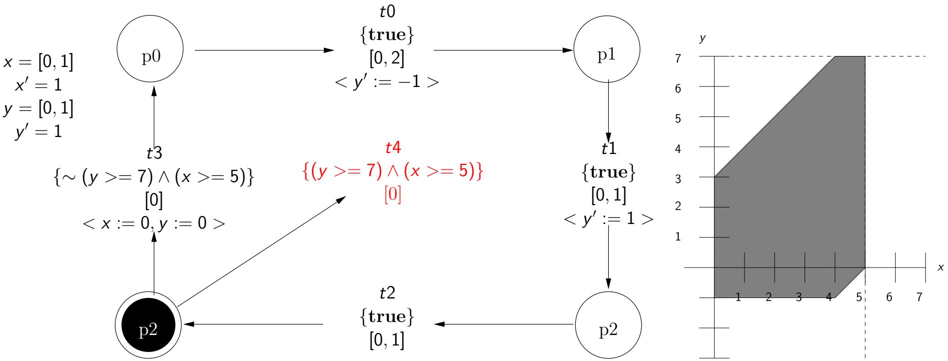
Negative Zone Warping: False Negative



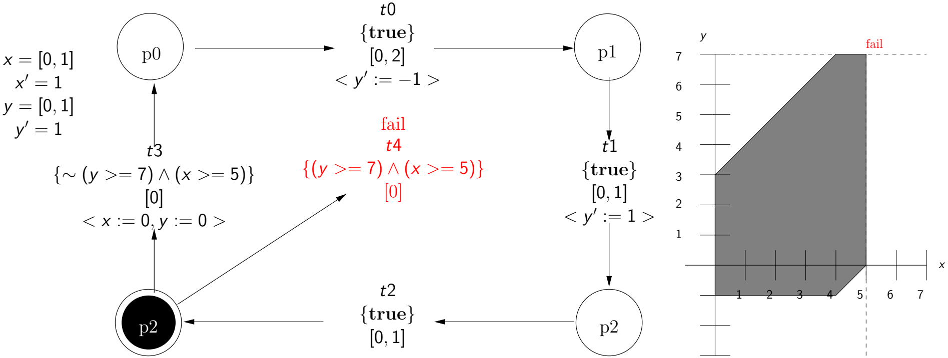
Negative Zone Warping: False Negative



Negative Zone Warping: False Negative

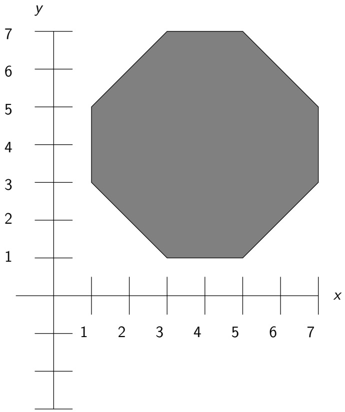


Negative Zone Warping: False Negative

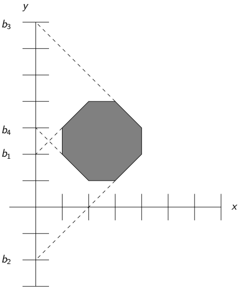


Octagons

- Extension of zones that allow negative 45° degree angles.



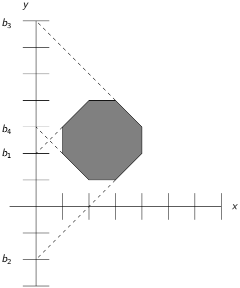
- Can be represented using a DBM (Mine, 2001) and manipulated with efficient algorithms.



$$
\begin{aligned}
x^+ - x^- &\leq 2M_x \\
x^- - x^+ &\leq -2m_x \\
y^+ - y^- &\leq 2M_y \\
y^- - y^- &\leq -2m_y
\end{aligned}
\qquad
\begin{array}{c}
\\ x^+ \\ x^- \\ y^+ \\ y^-
\end{array}
\begin{array}{c}
\begin{array}{cccc} x^+ & x^- & y^+ & y^- \end{array} \\
\begin{pmatrix}
0 & -2m_x & & \\
2M_x & 0 & & \\
 & & 0 & -2m_y \\
 & & 2M_y & 0
\end{pmatrix}
\end{array}
$$

Octagon DBM

- Can be represented using a DBM (Mine, 2001) and manipulated with efficient algorithms.



$$y^+ - x^+ \le b_1$$
$$x^- - y^- \le b_1$$
$$y^- - x^- \le -b_2$$
$$x^+ - y^+ \le -b_2$$
$$y^+ - x^- \le b_3$$
$$x^+ - y^- \le b_3$$
$$y^- - x^+ \le -b_4$$
$$x^- - y^+ \le -b_4$$

$$\begin{array}{c} \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array} \begin{array}{c} \begin{array}{cccc} x^+ & x^- & y^+ & y^- \end{array} \\ \begin{pmatrix} 0 & -2m_x & b_1 & -b_4 \\ 2M_x & 0 & b_3 & -b_2 \\ -b_2 & -b_4 & 0 & -2m_y \\ b_3 & b_1 & 2M_y & 0 \end{pmatrix} \end{array}$$

Reachability Analysis Using Octagons

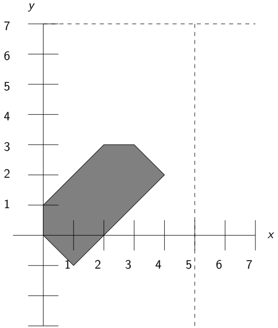
- Utilized for software checking, and efficient restriction, projection, and constraint tightening algorithms have been developed.
- New algorithms are needed to add new continuous variables, advance time, and warp the octagon.

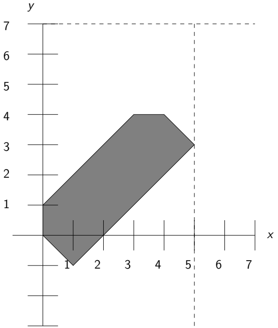
Adding Variables to Octagons

- Adding new continuous variables and clocks is simply a matter of re-interpreting the algorithms for zones in the language for octagons.
- When adding a continuous variable v with rate r, the maximum and minimum values for v are divided by r and added to the DBM (after multiplying by 2).
- Relational entries are set to infinity, indicating no relationship.

Octagon Time Advancement

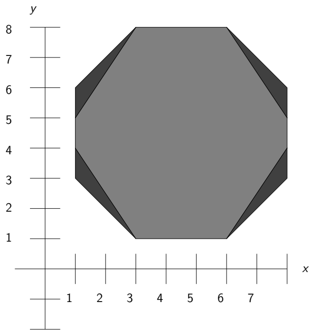
- Extend the octagon along the 45° lines.
- For zones, to advance time, simply set the upper bounds for all the variables to the maximum allowed value before an event occurs.
- For octagons, -45° line slicing the upper right hand corner has a limiting effect on the upper bounds of the two variables involved.
- Entries associated with inequalities $y + x \leq c$ must also be set to their maximum allowed value in relation to the maximums of x and y.



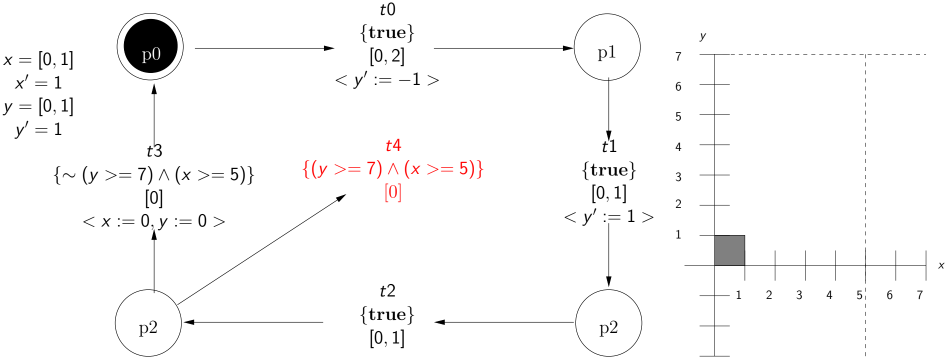


Octagon Warping

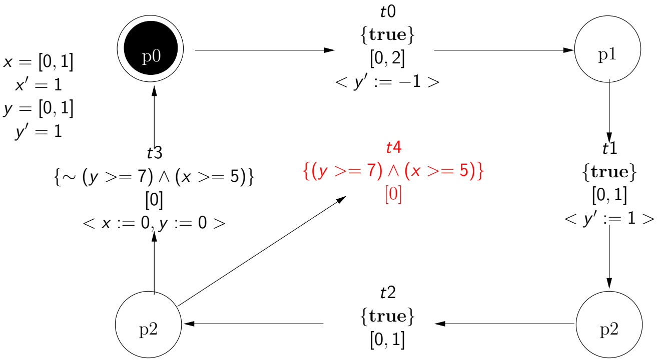
- Again replace every variable v by $\frac{v}{r}$ where r is the rate of v.
- Replace resulting polyhedra with smallest octagon that contains it.
- Accomplished by using a few algebraic equations that determine where the new axis intercepts are in terms of the old intercept values.

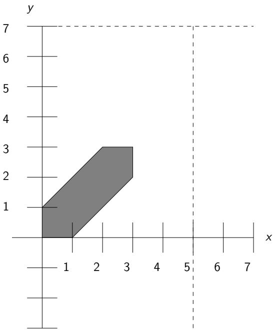


Octagon Example

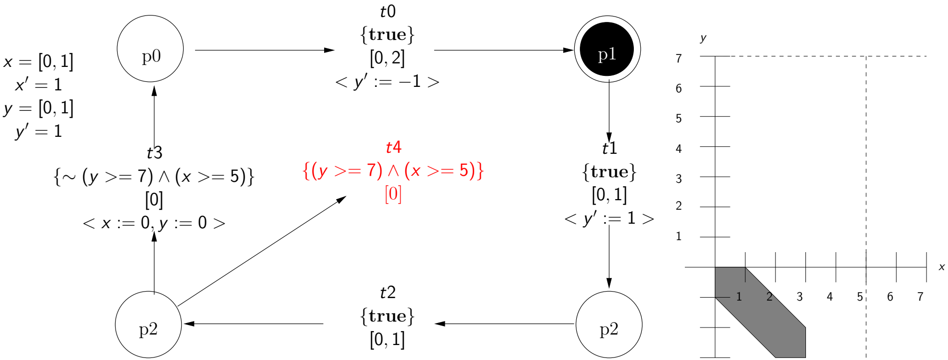


Octagon Example

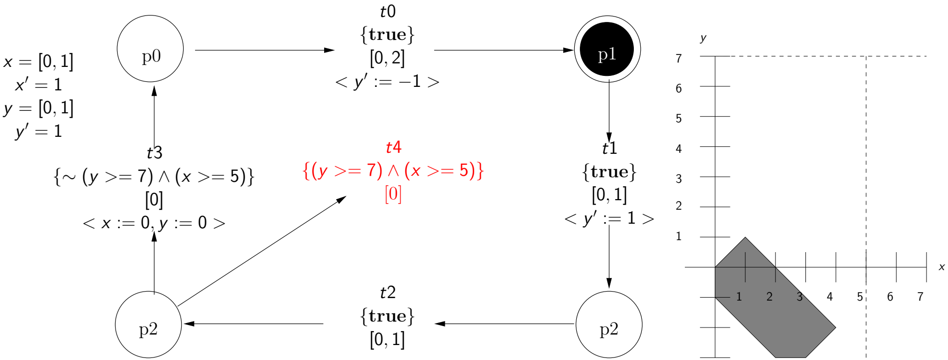




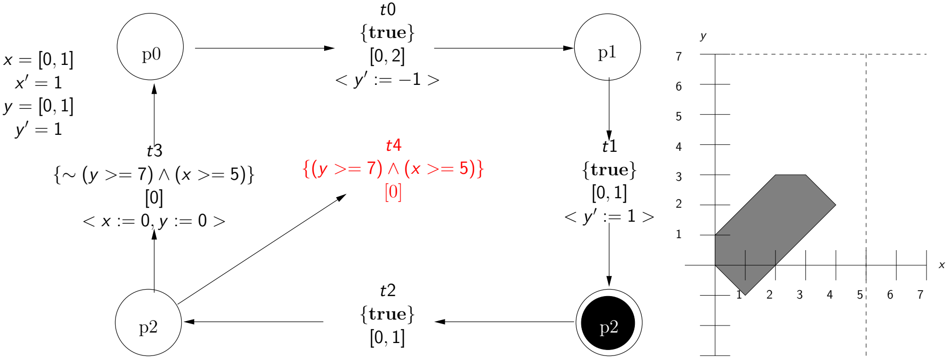
Octagon Example



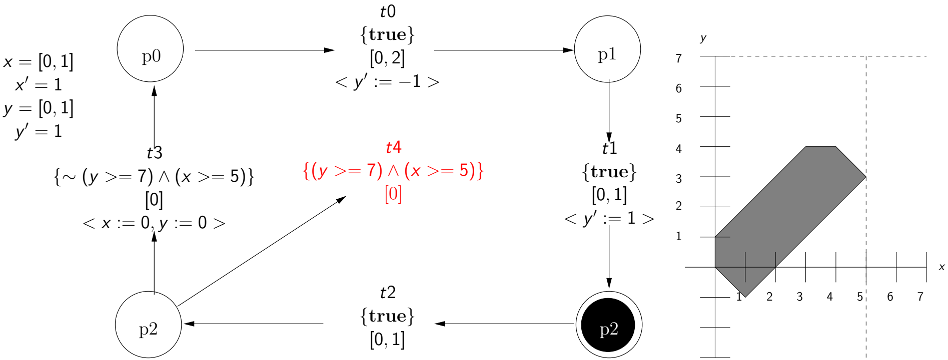
Octagon Example



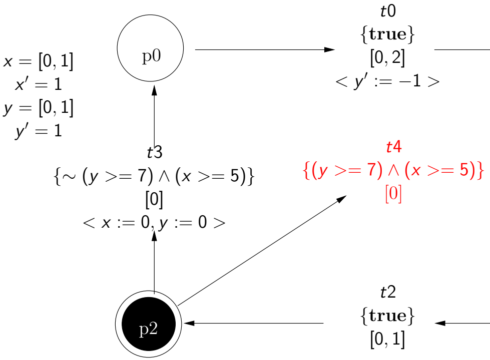
Octagon Example

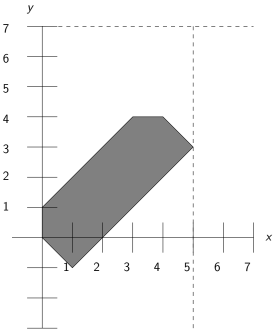


Octagon Example

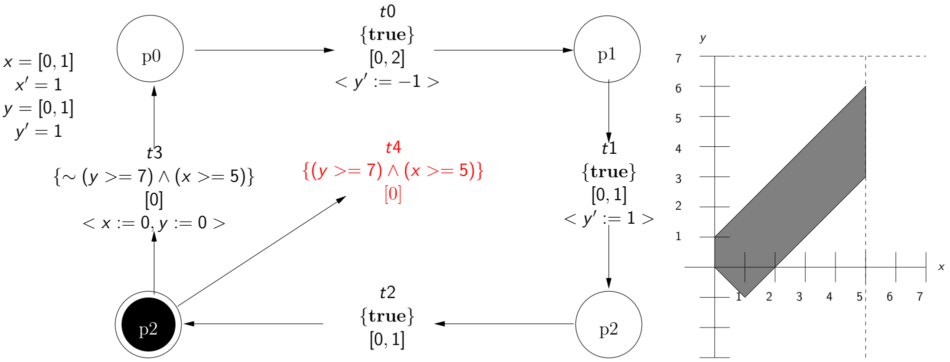


Octagon Example

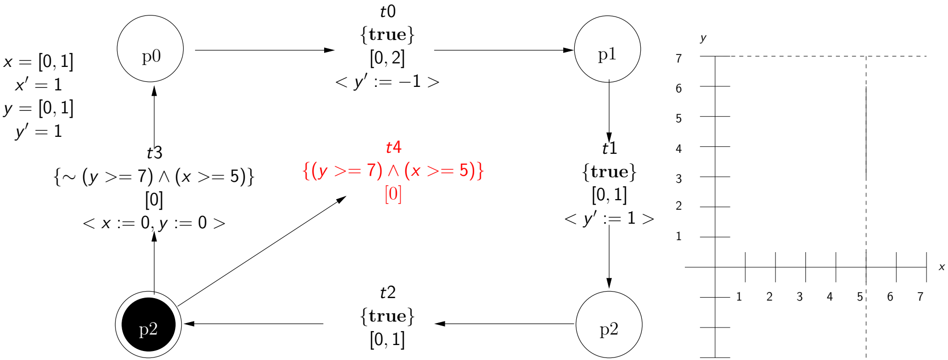




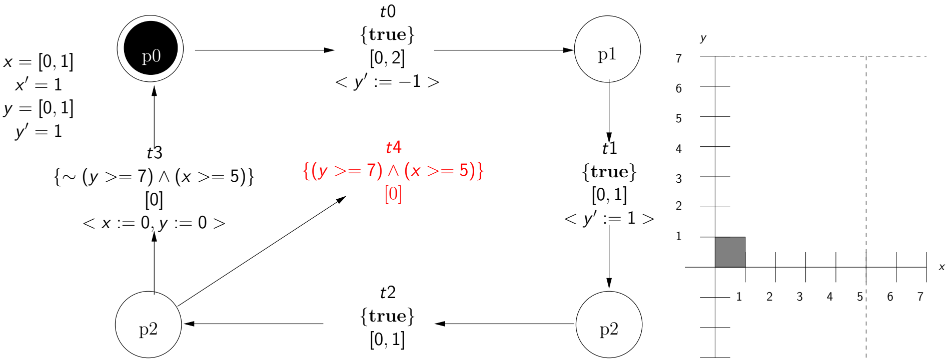
Octagon Example



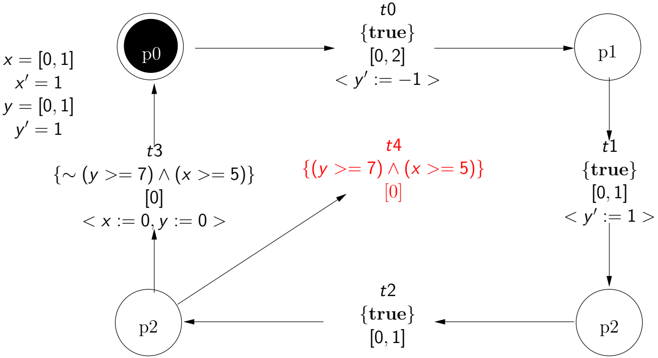
Octagon Example

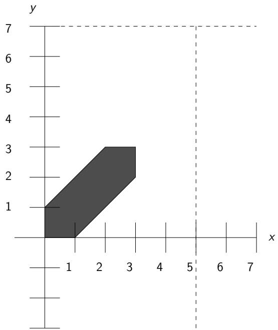


Octagon Example

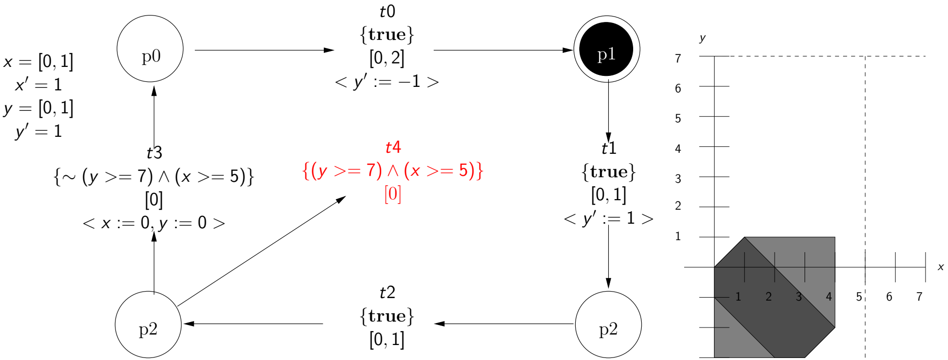


Comparison with Zones

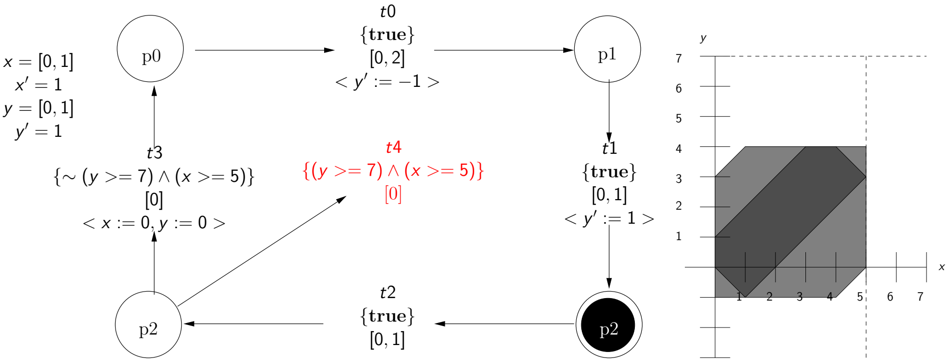


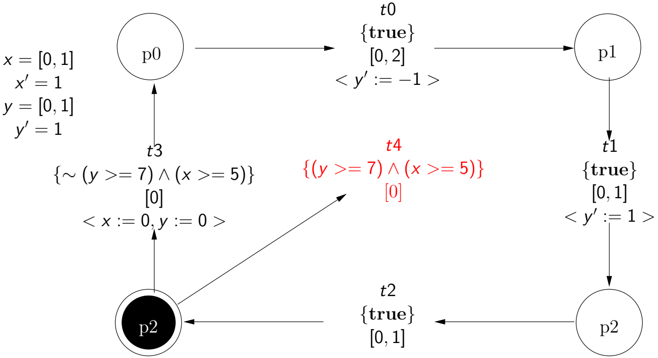


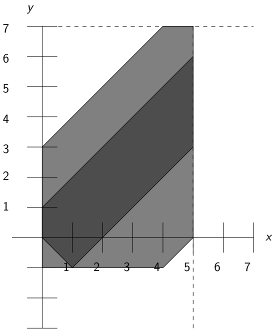
Comparison with Zones



Comparison with Zones



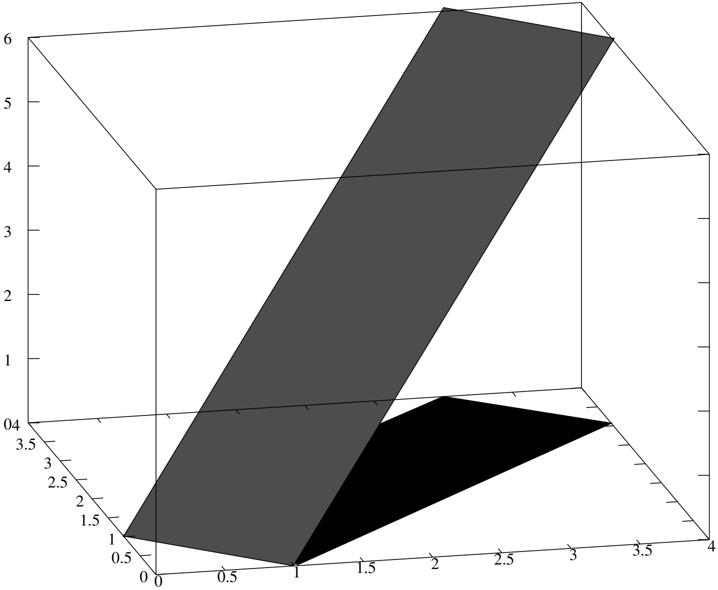

x = [0, 1]
x′ = 1
y = [0, 1]
y′ = 1
p0
t0
{true}
[0, 2]
< y′ := −1 >
p1
t1
{true}
[0, 1]
< y′ := 1 >
p2
t2
{true}
[0, 1]
p2
t3
{∼ (y >= 7) ∧ (x >= 5)}
[0]
< x := 0, y := 0 >
t4
{(y >= 7) ∧ (x >= 5)}
[0]


y
7
6
5
4
3
2
1
x
1
2
3
4
5
6
7

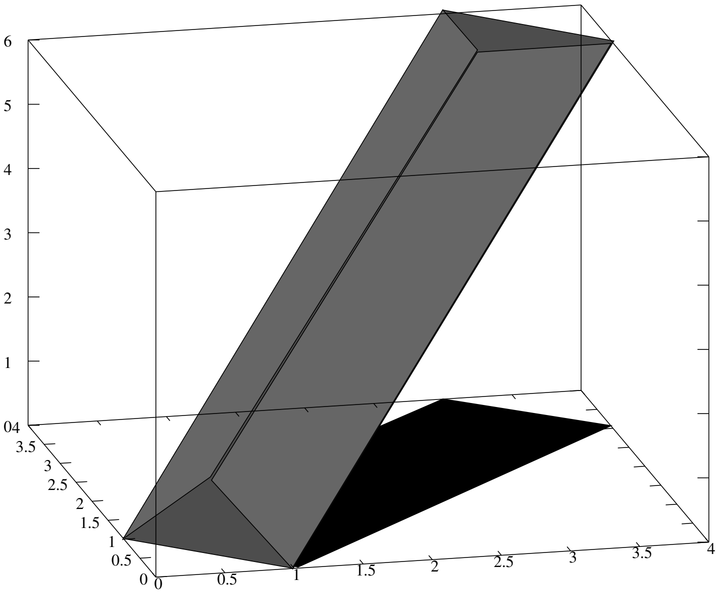
False Negatives

- Octagons do not eliminate the possibility of false negatives even in the case where rates are only ± 1.
- Time advancement also introduces a degree of over-approximation, related to the negative 45° lines.
- Advancement in three dimensions of one of these negative 45° line segments belongs to a plane of the form $ax + by + cz = d$.
- The bounding hyper-planes are of the form $\pm v_i \pm v_j \leq c$ and not able to capture this plane produced by advancing time.

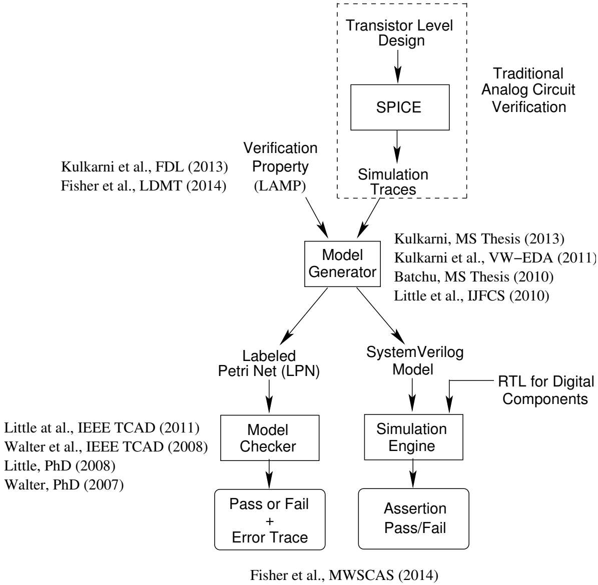
False Negative Example



False Negative Example



LEMA: LPN Embedded Mixed-Signal Analyzer



Acknowledgements

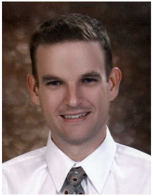
Satish Batchu (Qualcomm)

Andrew Fisher (Utah)

Kevin Jones (Aberdeen)

Dhanashree Kulkarni (Intel)

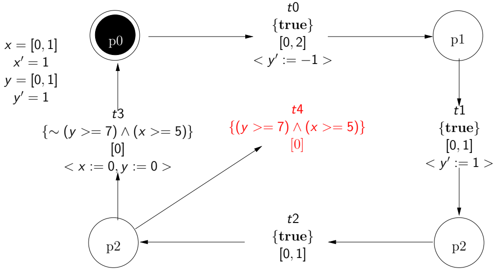
Scott Little (Intel)

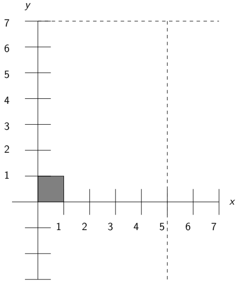


David Walter (Virginia State)

Supported by SRC Contracts 2002-TJ-1024, 2005-TJ-1357, 2008-TJ-1851, NSF Grant CCF-1117515, and by Intel Corporation.

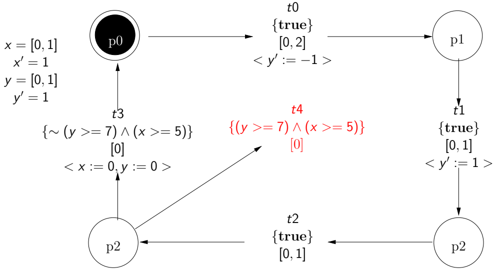
Octagon DBM

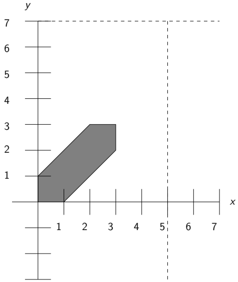




$$
\begin{array}{c}
 \\ t0^{+} \\ t0^{-} \\ x^{+} \\ x^{-} \\ y^{+} \\ y^{-}
\end{array}
\begin{array}{c}
\begin{array}{cccccc} t0^{+} & t0^{-} & x^{+} & x^{-} & y^{+} & y^{-} \end{array} \\
\begin{pmatrix}
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 2 & 0 & 2 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 2 & 1 & 2 & 0
\end{pmatrix}
\end{array}
$$

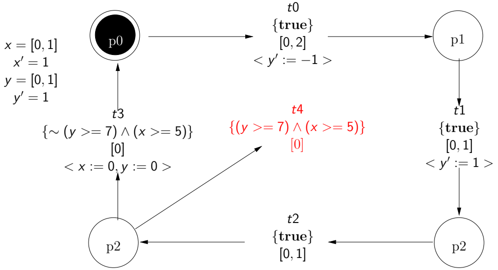
Octagon DBM

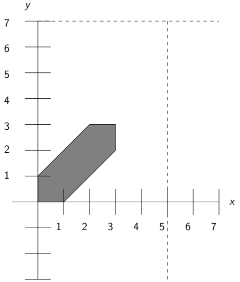




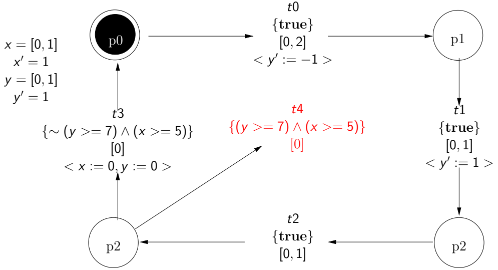
	$t0^+$	$t0^-$	x^+	x^-	y^+	y^-
$t0^+$	0	0	1	0	1	0
$t0^-$	4	0	1	0	1	0
x^+	0	0	0	0	1	0
x^-	1	1	10	0	2	1
y^+	0	0	1	0	0	0
y^-	1	1	2	1	14	0

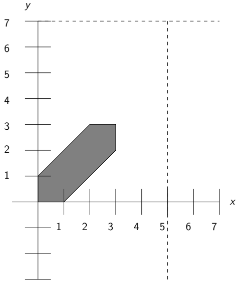
Octagon DBM





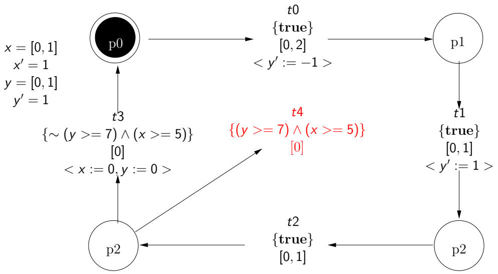
	$t0^+$	$t0^-$	x^+	x^-	y^+	y^-
$t0^+$	0	0	1	0	1	0
$t0^-$	4	0	1	0	1	0
x^+	0	0	0	0	1	0
x^-	1	1	10	0	∞	1
y^+	0	0	1	0	0	0
y^-	1	1	∞	1	14	0

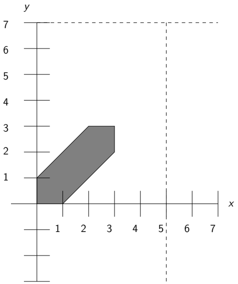




$$
\begin{array}{c|cccccc}
 & t0^+ & t0^- & x^+ & x^- & y^+ & y^- \\
t0^+ & 0 & 0 & 1 & 0 & 1 & 0 \\
t0^- & 4 & 0 & 1 & 0 & 1 & 0 \\
x^+ & 0 & 0 & 0 & 0 & 1 & 0 \\
x^- & 1 & 1 & 6 & 0 & 6 & 1 \\
y^+ & 0 & 0 & 1 & 0 & 0 & 0 \\
y^- & 1 & 1 & 6 & 1 & 6 & 0
\end{array}
$$

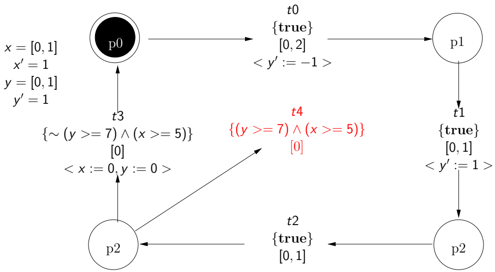
Octagon DBM

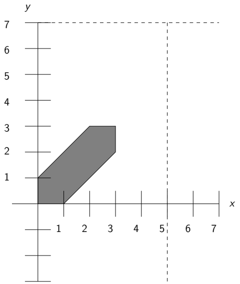




$$
\begin{array}{c} \\ t0^+ \\ t0^- \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array}
\begin{array}{c}
\begin{array}{cccccc} t0^+ & t0^- & x^+ & x^- & y^+ & y^- \end{array} \\
\begin{pmatrix}
0 & 0 & 1 & 0 & 1 & 0 \\
4 & 0 & \infty & 0 & \infty & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\infty & 1 & 6 & 0 & 6 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
\infty & 1 & 6 & 1 & 6 & 0
\end{pmatrix}
\end{array}
$$

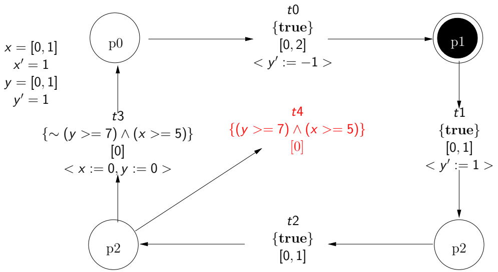
Octagon DBM

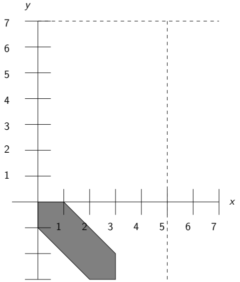




$$
\begin{array}{c} \\ t0^+ \\ t0^- \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array}
\begin{array}{c}
\begin{array}{cccccc} t0^+ & t0^- & x^+ & x^- & y^+ & y^- \end{array} \\
\begin{pmatrix}
0 & 0 & 1 & 0 & 1 & 0 \\
4 & 0 & 5 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
5 & 1 & 6 & 0 & 6 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
5 & 1 & 6 & 1 & 6 & 0
\end{pmatrix}
\end{array}
$$

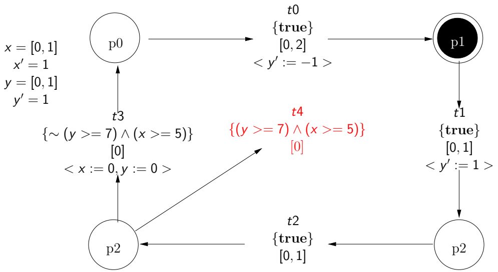
Octagon DBM

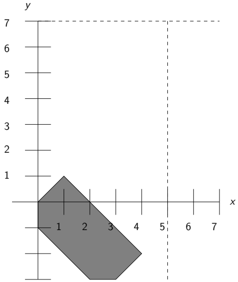




$$
\begin{array}{c|cccccc}
 & t1^+ & t1^- & x^+ & x^- & y^+ & y^- \\
\hline
t1^+ & 0 & 0 & 3 & 0 & 0 & 3 \\
t1^- & 0 & 0 & 3 & 0 & 0 & 3 \\
x^+ & 0 & 0 & 0 & 0 & 0 & 1 \\
x^- & 3 & 3 & 6 & 0 & 1 & 6 \\
y^+ & 3 & 3 & 6 & 1 & 0 & 6 \\
y^- & 0 & 0 & 1 & 0 & 0 & 0
\end{array}
$$

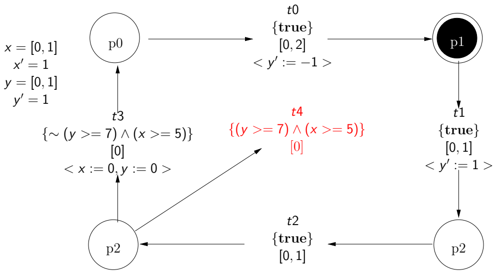
Octagon DBM

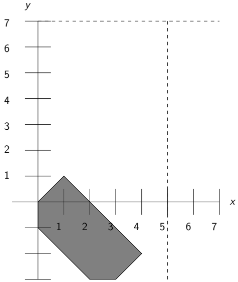




	$t1^+$	$t1^-$	x^+	x^-	y^+	y^-
$t1^+$	0	0	3	0	0	3
$t1^-$	2	0	3	0	0	3
x^+	0	0	0	0	0	1
x^-	3	3	10	0	1	6
y^+	3	3	6	1	0	6
y^-	0	0	1	0	14	0

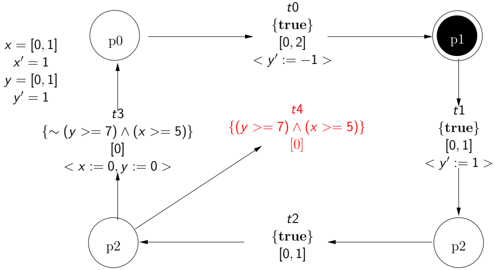
Octagon DBM

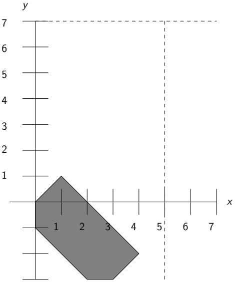




	$t1^+$	$t1^-$	x^+	x^-	y^+	y^-
$t1^+$	0	0	3	0	0	3
$t1^-$	2	0	∞	0	∞	3
x^+	0	0	0	0	0	1
x^-	∞	3	10	0	1	6
y^+	3	3	6	1	0	6
y^-	∞	0	1	0	14	0

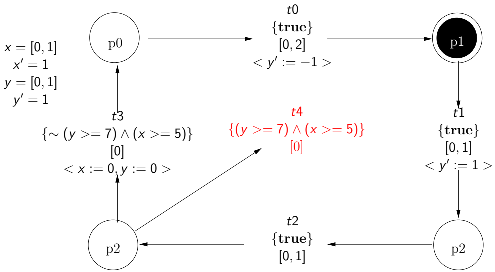
Octagon DBM

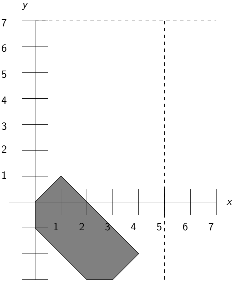




	$t1^+$	$t1^-$	x^+	x^-	y^+	y^-
$t1^+$	0	0	3	0	0	3
$t1^-$	2	0	∞	0	∞	3
x^+	0	0	0	0	0	1
x^-	∞	3	10	0	∞	6
y^+	3	3	6	1	0	6
y^-	∞	0	∞	0	14	0

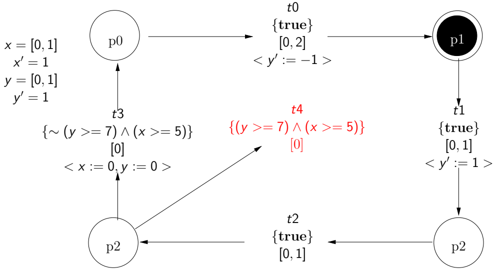
Octagon DBM

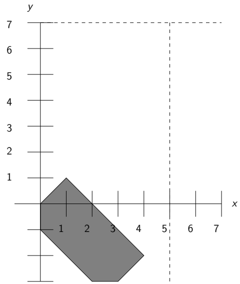




$$
\begin{array}{c} \\ t1^+ \\ t1^- \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array}
\begin{array}{c}
\begin{array}{cccccc} t1^+ & t1^- & x^+ & x^- & y^+ & y^- \end{array} \\
\begin{pmatrix}
0 & 0 & 3 & 0 & 0 & 3 \\
2 & 0 & \infty & 0 & \infty & 3 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\infty & 3 & 10 & 0 & 2 & 6 \\
3 & 3 & 6 & 1 & 0 & 6 \\
\infty & 0 & 2 & 0 & 14 & 0
\end{pmatrix}
\end{array}
$$

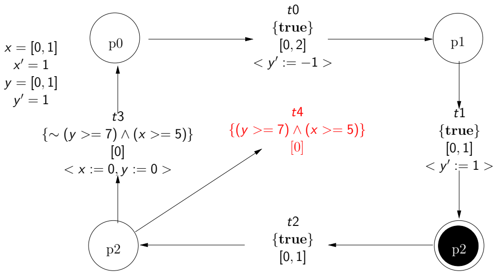
Octagon DBM

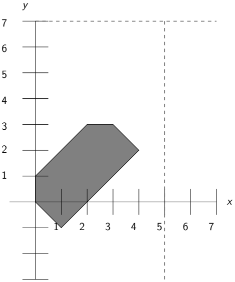




	$t1^+$	$t1^-$	x^+	x^-	y^+	y^-
$t1^+$	0	0	3	0	0	3
$t1^-$	2	0	5	0	2	3
x^+	0	0	0	0	0	1
x^-	5	3	8	0	2	6
y^+	3	3	6	1	0	6
y^-	2	0	2	0	2	0

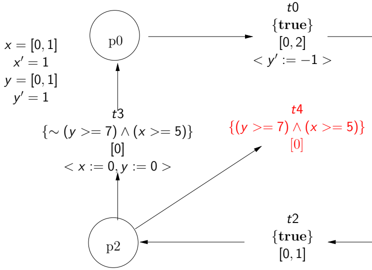
Octagon DBM

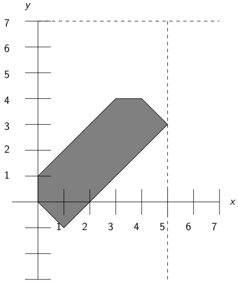




	$t2^+$	$t2^-$	x^+	x^-	y^+	y^-
$t2^+$	0	0	4	0	3	1
$t2^-$	0	0	4	0	3	1
x^+	0	0	0	0	1	0
x^-	4	4	8	0	6	2
y^+	1	1	2	0	0	2
y^-	3	3	6	1	6	0

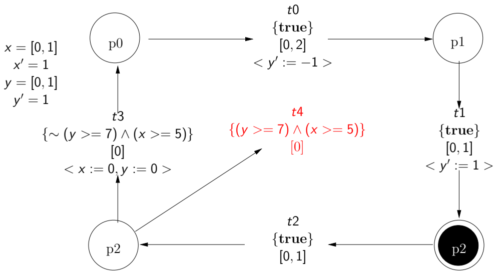
Octagon DBM

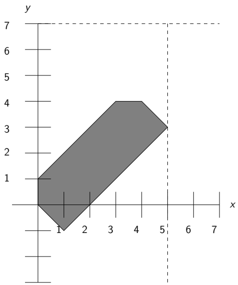




	$t2^+$	$t2^-$	x^+	x^-	y^+	y^-
$t2^+$	0	0	4	0	3	1
$t2^-$	2	0	∞	0	∞	1
x^+	0	0	0	0	1	0
x^-	∞	4	10	0	6	2
y^+	1	1	2	0	0	2
y^-	∞	3	6	1	14	0

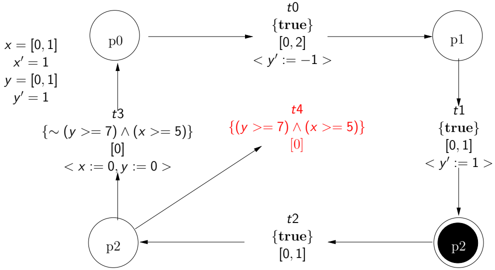
Octagon DBM

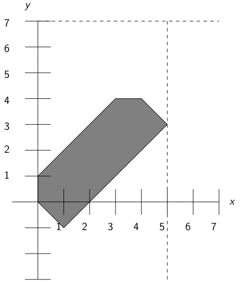




	$t2^+$	$t2^-$	x^+	x^-	y^+	y^-
$t2^+$	0	0	4	0	3	1
$t2^-$	2	0	∞	0	∞	1
x^+	0	0	0	0	1	0
x^-	∞	4	10	0	∞	2
y^+	1	1	2	0	0	2
y^-	∞	3	∞	1	14	0

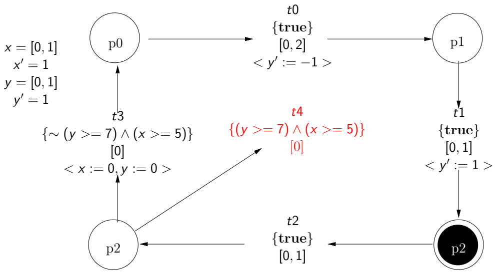
Octagon DBM

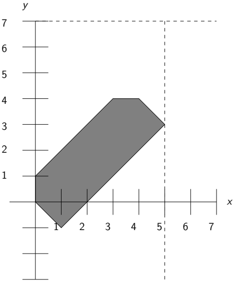




	$t2^+$	$t2^-$	x^+	x^-	y^+	y^-
$t2^+$	0	0	4	0	3	1
$t2^-$	2	0	∞	0	∞	1
x^+	0	0	0	0	1	0
x^-	∞	4	10	0	8	2
y^+	1	1	2	0	0	2
y^-	∞	3	8	1	14	0

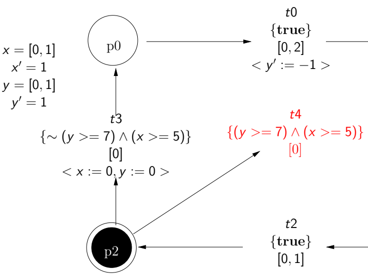
Octagon DBM

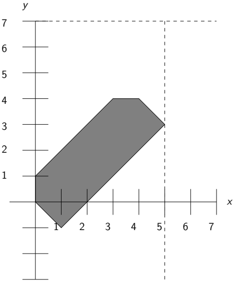




$$\begin{array}{c} \\ t2^+ \\ t2^- \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array}
\begin{array}{c} \begin{array}{cccccc} t2^+ & t2^- & x^+ & x^- & y^+ & y^- \end{array} \\
\left(\begin{array}{cccccc}
0 & 0 & 4 & 0 & 3 & 1 \\
2 & 0 & 6 & 0 & 5 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 \\
6 & 4 & 10 & 0 & 8 & 2 \\
1 & 1 & 2 & 0 & 0 & 2 \\
5 & 3 & 8 & 1 & 8 & 0
\end{array}\right) \end{array}$$

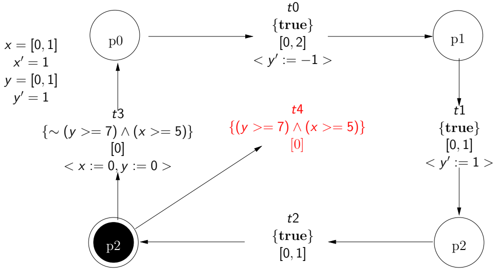
Octagon DBM

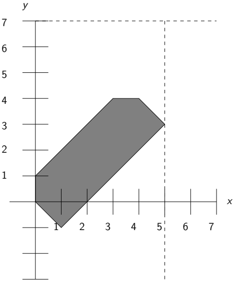




	$t3^+$	$t3^-$	x^+	x^-	y^+	y^-
$t3^+$	0	0	5	0	4	1
$t3^-$	0	0	5	0	4	1
x^+	0	0	0	0	1	0
x^-	5	5	10	0	8	2
y^+	1	1	2	0	0	2
y^-	4	4	8	1	8	0

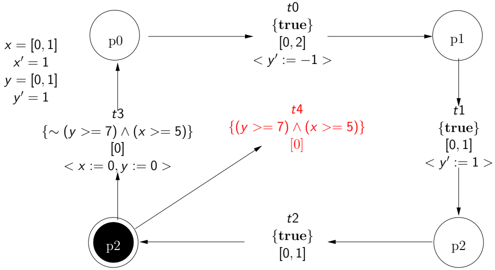
Octagon DBM

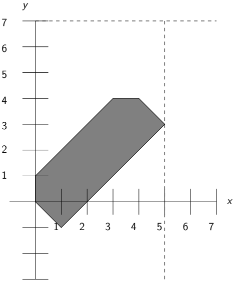




$$
\begin{array}{c} \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array}
\begin{array}{c}
\begin{array}{cccc} x^+ & x^- & y^+ & y^- \end{array} \\
\begin{pmatrix}
0 & 0 & 1 & 0 \\
10 & 0 & 8 & 2 \\
2 & 0 & 0 & 2 \\
8 & 1 & 8 & 0
\end{pmatrix}
\end{array}
$$

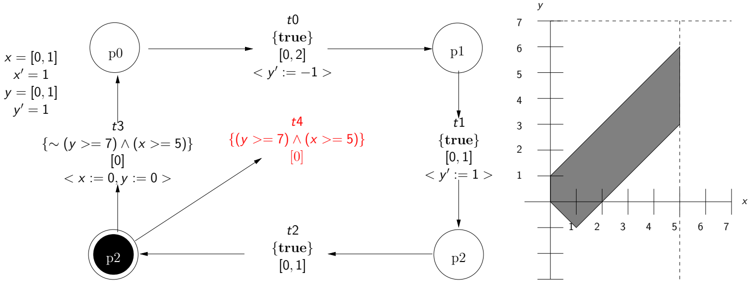
Octagon DBM



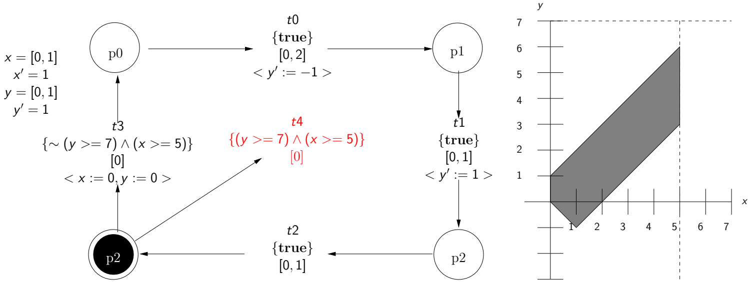


$$
\begin{array}{c} \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array}
\begin{array}{c} \begin{array}{cccc} x^+ & x^- & y^+ & y^- \end{array} \\
\begin{pmatrix} 0 & 0 & 1 & 0 \\ 10 & 0 & 8 & 2 \\ 2 & 0 & 0 & 2 \\ 8 & 1 & 14 & 0 \end{pmatrix} \end{array}
$$

Octagon DBM

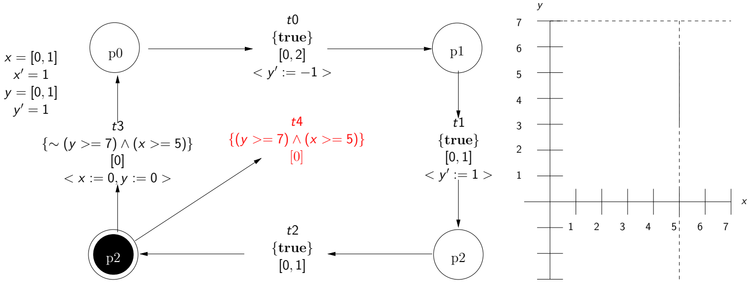


$$
\begin{array}{c} \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array}
\begin{array}{c}
\begin{array}{cccc} x^+ & x^- & y^+ & y^- \end{array} \\
\begin{pmatrix} 0 & 0 & 1 & 0 \\ 10 & 0 & \infty & 2 \\ 2 & 0 & 0 & 2 \\ \infty & 1 & 14 & 0 \end{pmatrix}
\end{array}
$$



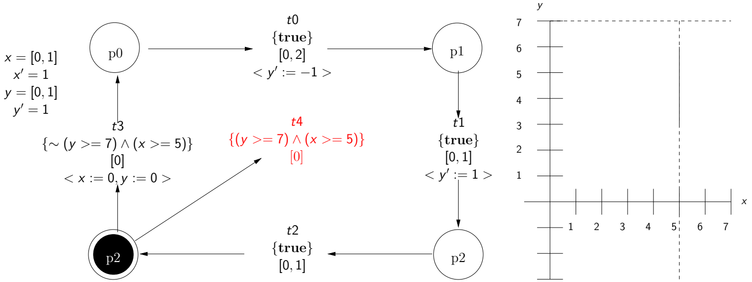
$$
\begin{array}{c c}
 & \begin{array}{cccc} x^+ & x^- & y^+ & y^- \end{array} \\
\begin{array}{c} x^+ \\ x^- \\ y^+ \\ y^- \end{array} &
\begin{pmatrix} 0 & 0 & 1 & 0 \\ 10 & 0 & 11 & 2 \\ 2 & 0 & 0 & 2 \\ 11 & 1 & 14 & 0 \end{pmatrix}
\end{array}
$$

Octagon DBM



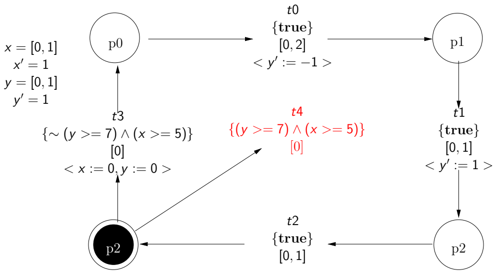
$$
\begin{array}{c} \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array}
\begin{array}{c}
\begin{array}{cccc} x^+ & x^- & y^+ & y^- \end{array} \\
\begin{pmatrix}
0 & -10 & 1 & 0 \\
10 & 0 & 11 & 2 \\
2 & 0 & 0 & 2 \\
11 & 1 & 14 & 0
\end{pmatrix}
\end{array}
$$

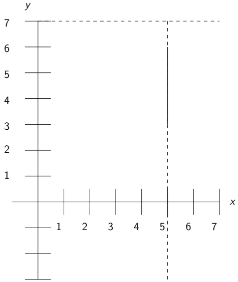
Octagon DBM



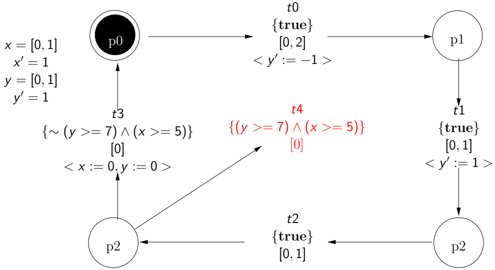
$$\begin{array}{c} \\ x^+ \\ x^- \\ y^+ \\ y^- \end{array} \begin{array}{c} \begin{array}{cccc} x^+ & x^- & y^+ & y^- \end{array} \\ \begin{pmatrix} 0 & -10 & 1 & -8 \\ 10 & 0 & 11 & 2 \\ 2 & -8 & 0 & 2 \\ 11 & 1 & 14 & 0 \end{pmatrix} \end{array}$$

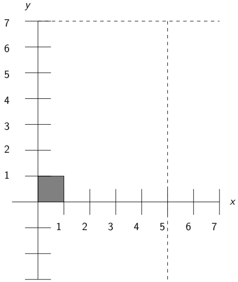
Octagon DBM





	$t3^+$	$t3^-$	x^+	x^-	y^+	y^-
$t3^+$	0	0	5	−5	6	−3
$t3^-$	0	0	5	−5	6	−3
x^+	−5	−5	0	−10	1	−8
x^-	5	5	10	0	11	2
y^+	−3	−3	2	−8	0	2
y^-	3	3	11	1	14	0





	$t0^+$	$t0^-$	x^+	x^-	y^+	y^-
$t0^+$	0	0	1	0	1	0
$t0^-$	0	0	1	0	1	0
x^+	0	0	0	0	1	0
x^-	1	1	2	0	2	1
y^+	0	0	1	0	0	0
y^-	1	1	2	1	2	0