

Automatic Modeling of Transistor Level Circuits by Hybrid Systems with Parameter Variable Matrices

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Abstract—Model abstraction of transistor-level circuits, while preserving an accurate behavior, is still an open problem. In this paper an approach is presented that automatically generates a hybrid automaton (HA) with linear states. The resulting HA is used for reachability analysis. Each of the locations of the HA is modeled with a system matrix described as a matrix zonotope or interval matrix. This leads to an acceptable over approximation, but guarantees that the system behavior is covered. The approach starts with a netlist at transistor level with full SPICE accuracy and ends at the system level description of the circuit. To illustrate our methodology, an example of a circuit containing complex poles and strong nonlinear limiting behavior is analyzed.

Index Terms—abstraction, verification, hybrid automaton, reachability, behavioral modeling, parameter variable matrices

I. INTRODUCTION

The modern world increasingly relies on circuits controlling safety critical systems. The utilizations of these circuits range from autonomous driving [1], human-robot interactions, robotic surgery, up till applications in our own bodies such as in cardiopulmonary bypass and mechanical ventilation. One of the approaches that guarantees system safety is formal verification. Formal verification has deployed solid methodologies that can cope with growing complexity of digital systems.

In the analog or continuous domain the complexity due to theoretically infinite number of states and strong nonlinear dynamic behavior is a challenge formal verification has to overcome. Moreover, to close the gap from the continuous physical environment connected with analog interfacing circuits to the digital signal processing a substantial effort has to be invested. In this paper we want to introduce an essential concept for formally verifying analog interface circuitry by means of hybrid automata on top of an existing approach [2]. Compared with the previous approach, this approach deploys a method that covers the full behavior exhibited by nonlinear circuit at transistor level with full SPICE BSIM accuracy. This is realized by using zonotopes or intervals to describe the system matrices. The approach is illustrated along an example circuit containing complex poles and strong nonlinear limiting behavior. In general, complex conjugate pole-pairs are very often intentionally used in filters, e.g. in antialiasing filters for ADCs or signal conditioning filters for sensor or actor signals.

A. Previous Work

Formal verification of analog transistor-level circuits [3] comes in equivalence checking [4], [5] and in model checking [6] flavors. However if we go up to higher levels, the behavior of the analog circuit has to be abstracted. In that area a long history in automatic modeling approaches [7] exist, all suffering from hardly being able to conduct the nonlinear behavior with the reduced order dynamics.

An advantage of an abstract model is its use in a powerful formal verification environment of hybrid systems [8]. There are some approaches that directly calculate nonlinear analog reduced order dynamics from transistor-level circuits [9]. In this paper we use the approach presented in [2] to generate directly a hybrid system using a sampling approach on a nonlinear analog transistor level circuit with full BSIM accuracy.

B. Overview

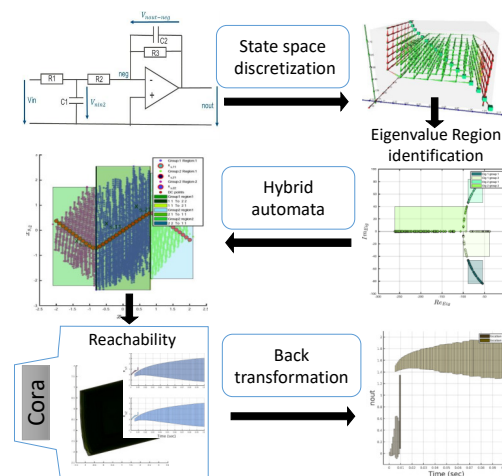


Fig. 1. Overview of the introduced methodology

An outline of the introduced approach is presented in Fig. 1. Starting from a SPICE netlist, both the original and the reduced state spaces are sampled. Based on the sampling of the reduced state space and by using the eigenvalues of the linearized system, the groups of the hybrid automaton (HA) are identified. Using a connection graph, regions of the HA are identified. The locations (states) of the HA are than

identified using the found groups and regions. At each location of the HA a linear system with a parameter variable system matrix describes the system behavior. The system matrices are found by hulling the eigenvalues at each location by zonotopes or intervals. Finally the HA is transformed into a *Cora* [8] supported syntax and a reachability analysis can be performed.

II. AUTOMATIC ABSTRACTION AND MODEL GENERATION

A. State Space Sampling

Using a state space sampler, the nonlinear transistor level circuit is linearized to a SISO system. Note that for ease of reading, we use a SISO system here. However the method is also able to handle MIMO systems. The sampling is done for each point in the state space of the nonlinear system and for all input voltages. This results in the following equation:

$$\begin{aligned} \underline{C} \cdot \Delta \dot{\vec{x}} + \underline{G} \cdot \Delta \vec{x} &= \vec{b} \cdot \Delta u \\ \Delta y &= \vec{r}^T \cdot \Delta \vec{x} \end{aligned} \quad (1)$$

with the capacitance matrix \underline{C} , the admittance matrix \underline{G} , and the input and output of the system u and y respectively. \vec{b} and \vec{r} are the input and output vectors. Note that the rank of \underline{C} represents the actual number of states (n) of the system. The system can be transformed from the original \vec{x} domain to a canonical state space \vec{x}_s , as described in [10], by multiplying state space equation with \underline{E} and using the transformation $\Delta \vec{x} = \underline{F} \Delta \vec{x}_s$. This yields the following equation:

$$\begin{aligned} s \cdot \underline{E} \cdot \underline{C} \cdot \underline{F} \cdot \Delta \vec{x}_s + \underline{E} \cdot \underline{G} \cdot \underline{F} \cdot \Delta \vec{x}_s &= \underline{E} \cdot \vec{b} \cdot \Delta u \\ \Delta y &= \vec{r}^T \cdot \underline{F} \cdot \Delta \vec{x}_s. \end{aligned} \quad (2)$$

Note that \underline{F} can be computed from the right eigenvectors of the generalized eigenproblem, while \underline{E} is a proper calculated matrix from the same problem. Expanding (2) and splitting the equation into a differential part subscript with (λ) with n equations and an algebraic part subscript (∞) with $\text{size}(\underline{C}) - n$ equations, (2) can be rewritten as:

$$\begin{aligned} s \cdot \begin{bmatrix} \underline{I} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} \begin{bmatrix} \Delta \vec{x}_{s,\lambda} \\ \Delta \vec{x}_{s,\infty} \end{bmatrix} + \begin{bmatrix} -\underline{\Lambda} & \underline{0} \\ \underline{0} & \underline{I} \end{bmatrix} \begin{bmatrix} \Delta \vec{x}_{s,\lambda} \\ \Delta \vec{x}_{s,\infty} \end{bmatrix} &= \begin{bmatrix} \vec{b}_\lambda \\ \vec{b}_\infty \end{bmatrix} \cdot \Delta u \\ \Delta y &= \begin{bmatrix} \vec{r}_\lambda^T & \vec{r}_\infty^T \end{bmatrix} \begin{bmatrix} \Delta \vec{x}_{s,\lambda} \\ \Delta \vec{x}_{s,\infty} \end{bmatrix} \end{aligned} \quad (3)$$

with:

$$\underline{F} = [\underline{F}_\lambda \underline{F}_\infty] \quad \underline{E} = \begin{bmatrix} \underline{E}_\lambda \\ \underline{E}_\infty \end{bmatrix} \quad (4)$$

Where \underline{I} is a identity matrix and $\underline{\Lambda}$ a diagonal or band-diagonal matrix filled with the finite eigenvalues from the underlying generalized Eigenproblem. Note that transformed vectors are marked with a tilde ($\vec{\cdot}$). The differential part of (3), subscript with λ , is referred to as the reduced state space of the system.

B. System Description in the Reduced State Space

With the data set at hand, containing both spaces (\vec{x} and \vec{x}_s), the groups and regions needed for the generation of the HA can be identified. As stated in [2], the group identification is done based on eigenvalues clustering, while the region identification

is done by using a connection graph. Note that regions are used to divide the groups if the points in the state space have similar eigenvalues but are reached with different input voltages. In order to describe the system behavior at each of the location of the HA, the eigenvalues and the steady state points (DC) are used. For each locations of the HA, the system is linearized around a suitable operating point. Each of the locations of the HA can then be described as:

$$\Delta \dot{\vec{x}}_s = \underline{A} \cdot \Delta \vec{x}_s + \underline{B} \Delta u \quad (5)$$

such that:

$$\begin{aligned} \Delta \vec{x}_s &= \vec{x}_s - \vec{x}_{DC} & \underline{A} &= -\underline{E}_\lambda \cdot \underline{G}_\lambda \cdot \underline{F}_\lambda = \underline{\Lambda} \\ \Delta u &= u - u_{DC} & \underline{B} &= \underline{E}_\lambda \cdot \vec{b} \end{aligned} \quad (6)$$

Instead of using the matrix \underline{A} containing the mean of the eigenvalues clustered, this approach utilizes a system matrix bounded by a matrix zonotope $\underline{A}_{[z]}$ or an interval matrix $\underline{A}_{[i]}$ as defined in [8]:

$$\underline{A}_{[z]} = \{ \underline{G}^{(0)} + \sum_{i=1}^k p_i \cdot \underline{G}^{(i)} \mid p_i \in [-1, 1], \underline{G}^{(i)} \in \mathbb{R}^{n \times n} \} \quad (7)$$

$$\underline{A}_{[i]} = [\underline{A}, \bar{\underline{A}}], \forall i, j : \underline{a}_{i,j} \leq \bar{a}_{i,j}, \underline{A}, \bar{\underline{A}} \in \mathbb{R}^{n \times n} \quad (8)$$

The geometric bounds of the system matrix \underline{A} can be identified by the eigenvalues. In order to illustrate the process, consider the circuit shown in Fig.2.

The circuit shown in Fig. 2 exhibits a strong nonlinear limiting

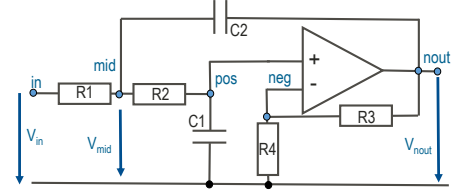


Fig. 2. Second order low-pass filter with a gain of 1.695, a complex pole pair, and a nonlinear limitation at 1.5V. The operational amplifier is a SPICE file consisting of 11 transistors in a 350nm CMOS technology [11].

behavior. Moreover, the eigenvalues of the linearized system obtained by means of equations (1) and (2) change from complex to purely real in the limiting regions. Fig. 3 shows the root locus of the system. The movement and conversion of the complex pole pair to two real poles due to the nonlinear behavior in the limiting region is clearly visible. Our approach describes a group of eigenvalues with an interval matrix as illustrated in Fig. 3.

The region identification is illustrated in Fig. 4. As seen, 2 groups have been identified by the eigenvalue clustering. Group 2 has been divided into 2 regions, which is due to the fact that both regions have the nearly the same sets of eigenvalues, but can be reached with different input voltages. The large red dots shown in Fig. 4 represent the operating points at different input voltages. Note that selected points from this set are used to linearize the systems as shown in equations (5), (6). Compared to Fig. 3 which uses interval

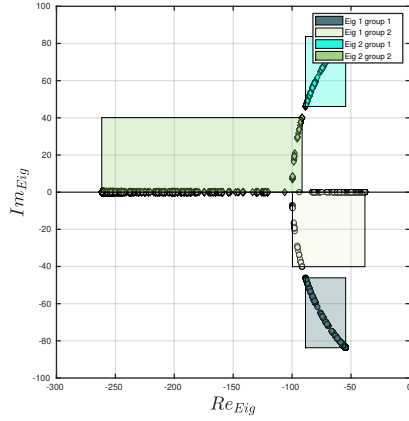


Fig. 3. Root locus of the circuit from Fig. 2. Each pair the colored intervals belonging to the same group represent the system matrix \underline{A}

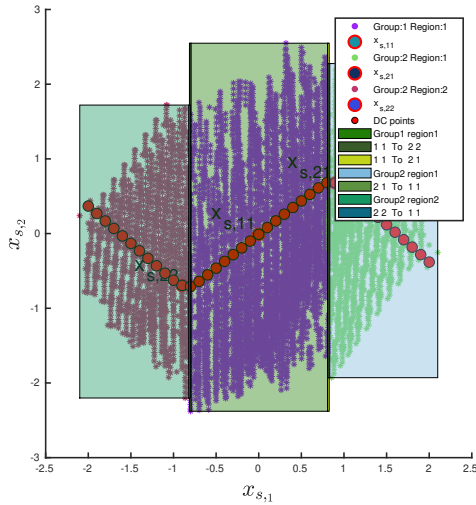


Fig. 4. State space of the reduced system of the circuit shown in Fig. 2

matrices to hull the eigenvalues, our approach works also with matrix zonotopes as is illustrated in Fig. 5. In fact Fig. 5 represents our methodology with 5 locations (left) and with 3 locations (right) for the HA, while Fig. 3 uses only 3 locations. Note that the zonotopic hull for some groups in Fig. 5 is too narrow to be drawn. Thus \underline{A} in (6) can now be replaced by a matrix zonotope or an interval matrix that hulls all eigenvalues at a location of the HA (λ_{loc}):

$$\underline{A}_{[z/i]} = \text{matZonotope}(\lambda_{loc}) / \text{matInterval}(\lambda_{loc})$$

C. Back-transformation into the \vec{x} -domain

After performing the reachability analysis on the HA, the result must be transformed back from the \vec{x}_s -domain into the \vec{x} -domain. This is done with following equation:

$$\Delta \vec{x} = \underline{C} \cdot \Delta \vec{x}_s + \underline{D} \Delta u \quad (9)$$

such that:

$$\Delta \vec{x} = \vec{x} - \vec{x}_{DC} \quad \underline{C} = \underline{F}_\lambda \quad \underline{D} = \underline{F}_\infty \cdot \underline{E}_\infty \cdot \vec{b} \quad (10)$$

Note that each location of the HA contains different values in (6) and (10), and that \vec{x} , \vec{x}_s and u from (9) represent

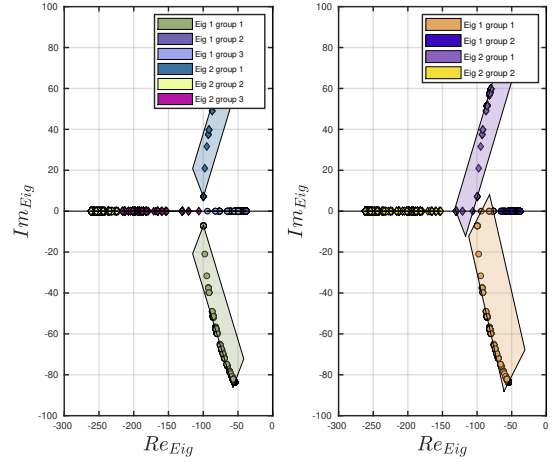


Fig. 5. Root locus of the circuit from Fig. 2. The right side of the image illustrates a HA with 3 location (11 21 22) while the left side illustrates a HA with 5 locations (11 21 22 31 32). The numbers represent the groups followed by the regions.

zonotopes. The matrices \underline{F} and \underline{E} are still matrices and not matrix zonotopes. In the future, these matrices will be replaced by matrix zonotopes to over approximate the system behavior. For the circuit in Fig. 7, this can be neglected as the changes in \underline{F} and \underline{E} are negligible within a location of the HA.

III. EXPERIMENTAL RESULTS

For the circuit from Fig. 2, the results of the reachability analysis with $V_{in} = 2$ V along with a back-transformation are illustrated in Fig. 6 for the output voltage V_{nout} . As seen, the HA described with matrix zonotopes (upper part of the image) exhibits better results than the HA described with interval matrices (middle part). The lower part of the image shows a HA with 5 locations described with interval matrices. As seen, this HA exhibits the best results but over approximates compared to the zonotope matrix with a lower number of locations. The discontinuity in the top and middle images and the behavior of the HA with 5 locations at G3R1 are the result of grouping all eigenvalues. The eigenvalues far to the left might have been the result of some numeric noise.

Consider the example from [2] shown in Fig. 7. The circuit exhibits a strong limiting behavior at 1.5V. Compared to the previous low-pass filter (Fig. 2), the circuit has only eigenvalues that change along the real axis. Additionally an uncertainty of 0.5V in the input voltage is considered. The reachability analysis of the reduced system is shown in Fig. 9, while Fig. 8 shows one of the 24 nodes from the \vec{x} -space (V_{nout}) versus time. Two reachability analyses have been performed with $V_{in} = 2$ V and $V_{in} = 3$ V.

IV. CONCLUSION

In this paper, an approach for the formal verification of transistor-level circuits using automatic abstraction to hybrid automaton has been proposed. Compared with the approach in [2], this approach utilizes the eigenvalues of the sampled state space and generates parameter variable system matrices

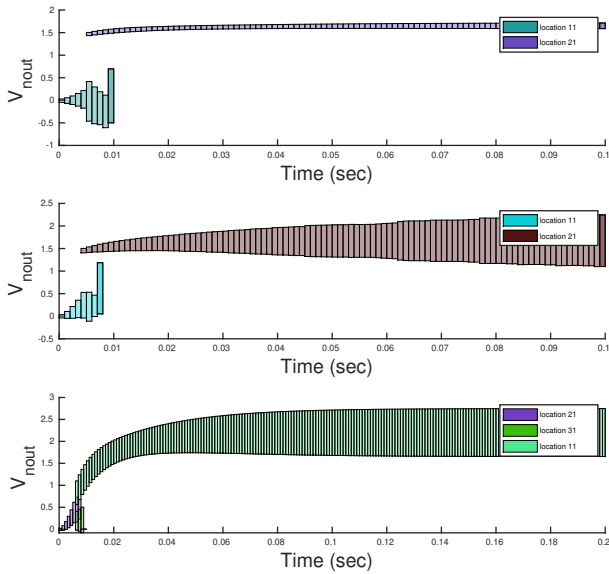


Fig. 6. V_{nout} from Fig. 2 versus time. Simulation is done with $V_{in} = 2V$. The upper part of the figure belongs to a HA described with matrix zonotopes with 3 locations (right side of Fig. 5), while the middle and lower part belongs to a HA described with interval matrices. Note that the middle image belong to a HA with 3 locations while the lower one belongs to a HA with 5 locations

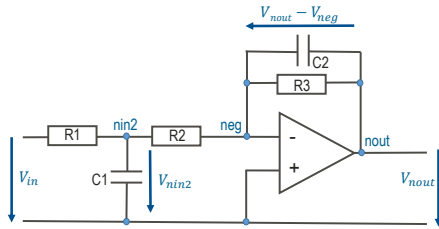


Fig. 7. Circuit of a second order low-pass filter from [2]. The corresponding netlist of the operational amplifier is a SPICE file consisting of 11 transistors in a 350nm CMOS technology [11].

that over-approximate the reachable region. Different settings can be applied to this abstraction resulting in either more abstract or more precise models and reachable regions. Future work will be done on an error measures and solving the discontinuities in the reachable regions. As seen throughout the paper, this approach shows promising results for the formal verification of a complex nonlinear system.

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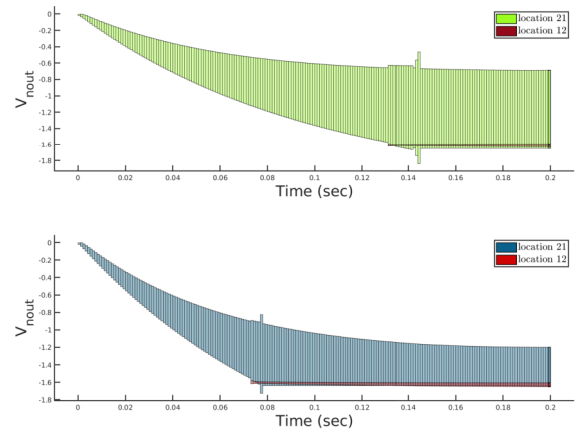


Fig. 8. V_{nout} from Fig. 7 versus time. Since the circuit shown in Fig. 7 has only real eigenvalues, no overshoot can be observed. The reachability analysis has been done with $V_{in} = 2 \pm 0.5 V$ (upper part) and $V_{in} = 3 \pm 0.5 V$ (lower part).

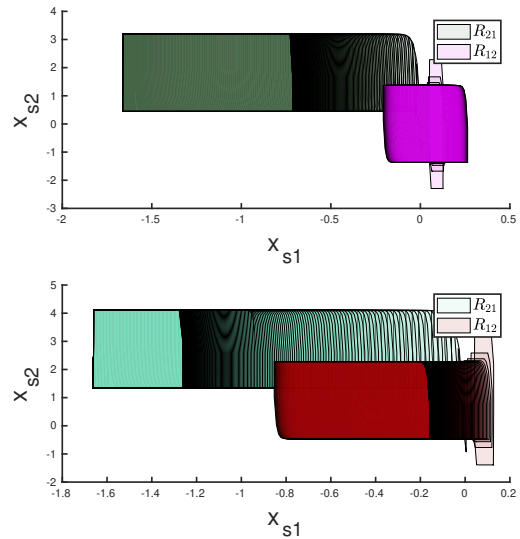


Fig. 9. Reachability analysis on the reduced state space of the circuit shown in Fig. 7. Note that the upper part of the figure has an input voltage $V_{in} = 2V$, while the lower part of the figure has an input voltage $V_{in} = 3V$.

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